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Decentralized Robust Control Strategies with Model Based Feedforward for Elastic Web Winding Systems

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Abstract— **Demand for improved performance under a wide variety of dynamic conditions and web materials are placing additional emphasis on developing new advanced control strategies. This paper presents decentralized $\mathcal{H}_\infty$ controllers with model based feedforward for web winding systems which provide improved web tension and velocity regulation in the presence of uncertainties. First, mathematical models of fundamental elements in a web process line are presented. A new state space model is developed which enables calculation of the phenomenological model feedforward signals and helps in the synthesis of decentralized $\mathcal{H}_\infty$ controllers around the set points given by the reference signals. Different $\mathcal{H}_\infty$ control strategies with additive feedforward have been validated on a nonlinear simulator identified on a 3-motor winding test bench.**

I. Introduction

THE systems handling web material such as textile, paper, polymer or metal are very common in the industry. Modeling and control of web handling systems have been studied for several decades. However, increasing requirement on control performance and better handling of elastic web material have led to search for more sophisticated robust control strategies. One of the objectives in such systems is to improve decoupling between web tension and speed, so that constant web tension can be maintained during process speed changes. So far, many industrial web transport systems have used decentralized PI-type controllers. However, for better control performance and robustness to uncertainties, more efficient control strategies such as LQG or $\mathcal{H}_\infty$ should be used. Most modern control law designs require the construction and validation of a reasonable plant model. In this article, we use a 3-motor nonlinear simulation model resulting from modeling and identification of an experimental bench. The detailed description of the nonlinear model is given in [14, 13]. The model of a large scale web winding system is then deduced from the experimentally verified model. A new state space description has been elaborated and is presented in the Appendix.

Robust control has already been applied to web handling for reduced-size systems, containing not more than 3 motors, with multivariable $\mathcal{H}_\infty$ centralized controllers and LPV structures [14]. Nevertheless, an industrial web process line is a large-scale interconnected system with a large number of control and measurement variables and it is not suitable to use a centralized controller for such process. Typically, a web process line is divided into a number of functional subsystems which include the unwind, rewind, and intermediate process sections. Decentralized control strategies provide an effective and simple means to control the entire process line by developing controllers for each subsystem using locally available informations. Therefore, a solution is to use semi-decentralized control: the global system is split into several subsystems controlled independently by its own controller [12, 2, 5].
Recently, multivariable decentralized control strategies have been proposed for industrial metal transport systems [8, 9], and for elastic web with $\mathcal{H}_\infty$ controllers [7, 12]. Decentralized control with overlapping of adjacent subsystems [15, 12, 3, 25], can be useful to reduce the coupling between two such subsystems. Such a control strategy has already given good results in the case of a vehicle platoon [16]. Several improvements of multivariable $\mathcal{H}_\infty$ controllers with one or two degrees of freedom [12, 2] and $\mathcal{H}_\infty$ state feedback control with full or partial integral actions [3, 4] have been applied to web winding systems.

A number of issues related to modeling and control of continuous web processing lines were investigated in [18, 19, 20]. Attenuation of periodic web tension disturbances using active dancer mechanisms was investigated in [21, 22]. A new model of the unwind/rewind roll that explicitly includes the time-varying inertia and radius of the roll was developed in [23]; based on the new model, a method for calculating feedforward inputs to keep the system at forced equilibrium of the reference web tension and velocity was given and a linearized model was obtained. A stable, decentralized state feedback controller was developed and experimentally verified on a large experimental web

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processing line. A decentralized adaptive controller with model based feedforward was developed and experimentally verified in [24].

This paper presents, for the first time, $\mathcal{H}_\infty$ control strategies coupled with model based feedforward control, applied to web transport systems. It is shown that such a strategy leads to much improved web tension and velocity regulation performance. The outline of the remainder of the paper is as follows. Section II gives the main physical laws used to build a nonlinear model which was also identified on an experimental bench composed of three motors (Fig. 1). Linearization of the model around a fixed web tension and velocity reference values gives the state space model that is useful for modern controller synthesis. Section III is dedicated to centralized and semi-centralized $\mathcal{H}_\infty$ control design with physical model based feedforward. A complete decentralized approach is then described in section IV. Finally, section V gives conclusions of this work and indicates some future research directions.

II. Plant Modeling

The nonlinear model [14] of a web transport system is built from the equations describing the web tension behavior between two consecutive rolls and the velocity of each roll. This model was identified on a 3-motor experimental bench given in Fig. 1.

Fig. 1. Experimental setup with 3 brushless motors and 2 load cells

A. Web Tension Calculation

The computation of the tensions between two rolls of web transport systems is based on three laws.

1) Hooke's law:

The tension T of an elastic web is a function of the web strain ε:

$$T = E\,S\,\varepsilon = E\,S\,(L - L_0)/L_0\ , \qquad (1)$$

where E is the modulus of elasticity, S the web cross section, L and L_0 are stretched and unstretched web lengthes, respectively.

2) Coulomb's law:

The study of a web tension on a roll can be considered as a problem of friction between solids [13].

3) Equation of Continuity:

This equation, applied to the web, yields [13]:

$$L\frac{dT_{k+1}}{dt} = E\,S(V_{k+1} - V_k) + T_k V_k - T_{k+1}(2V_k - V_{k+1})\ . \qquad (2)$$

where k is the span number and takes values from 0 to N-1.

B. Web Velocity Calculation

The linear velocity V_k of roll k is obtained from the torque balance [13]:

$$\frac{d}{dt}\left(J_k\frac{V_k}{R_k}\right) = R_k(T_k - T_{k-1}) + K_k U_k + C_f\ , \qquad (3)$$

where $K_k U_k$ is the motor torque and C_f is the friction torque. The inertia J_k and the radius R_k of unwind and winder are time dependent and vary substantially during processing.

A large scale web handling system of any number of driven rolls can be built from the equations (1), (2) and (3). A schematic representation of a multi-motor transport system is shown in Fig. 13 in the Appendix.

C. State Space Representation

A scheme of a 3-motor setup with PI controllers is represented in Fig. 2.

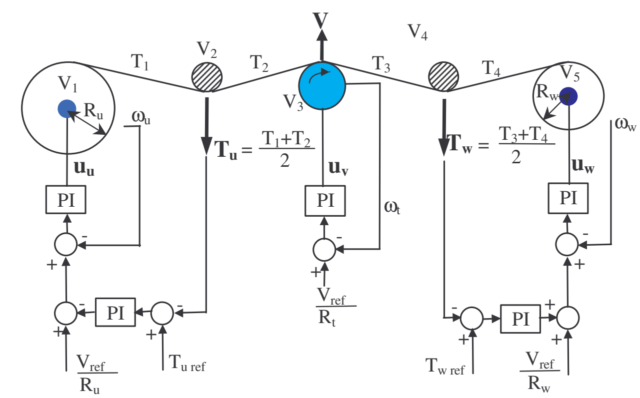

Fig. 2. Distributed industrial control scheme for a winding process

The inputs to the system are the torque control signals (u_u, u_v, u_w) of the brushless motors; the measurements are the unwinder and winder web tensions T_u and T_w and the web velocity $V = V_3$. The web velocity is set by the master traction motor whereas the web tensions in the spans are controlled by the unwind and rewind motors.

The nonlinear state-space model is composed of (2) for the different web spans and (3) for the different rolls. In [14] a global three-motor state space model is presented using a first order linearization and under the assumption that J_k/R_k is slowly varying. In this work, a more precise state space model is obtained by decomposing the nonlinear equations as follows: define

$$V_i = V_0 + v_i \qquad T_i = T_0 + t_i \qquad U_i = U_{si_0} + u_{si} \qquad (4)$$

where v_i, t_i, u_{si} are signal variations around the reference values.

The three-motor system variational dynamics can be presented in the following form:

Subsystem 1 (with $\underline{x}_1^T = [v_1 \quad t_1 \quad v_2 \quad t_2]$) :

$$E_1\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1 u_{s1} + H_1 + A_{12}\underline{x}_2 \qquad (5)$$

Subsystem 2 (with $\underline{x}_2^T = [v_3]$):

$$E_2 \dot{\underline{x}}_2 = A_2 \underline{x}_2 + B_2 u_{s2} + H_2 + A_{21} \underline{x}_1 + A_{23} \underline{x}_3 \quad (6)$$

Subsystem 3 (with $\underline{x}_3^T = [t_3 \quad v_4 \quad t_4 \quad v_5]$):

$$E_3 \dot{\underline{x}}_3 = A_3 \underline{x}_3 + B_3 u_{s3} + H_3 + A_{32} \underline{x}_2 \quad (7)$$

A detailed description of the first two subsystems is given in the appendix. The constant values and the non-linear terms are included in the H_i vectors whereas the matrices A_{ij} describe the effect (coupling) of subsystem j on subsystem i.

III. Centralized and Semi-Centralized Control with Feedforward for Winding Systems

The controller design for a 3-motor plant is the subject of the next subsection. An extension to a large scale system will also be indicated.

A. *Centralized H_∞ control design for a 3-Motor Plant*

Coupling between web velocity and tension makes the control of web transport systems inherently difficult. Several methods suppressing this coupling in a system with two driven rolls have been studied [25].

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbances rejection. The synthesis should be done using a linear model corresponding to the starting phase, i.e., an empty roller at the winder. The starting phase is very important: if a problem occurs in this phase, most likely, the rewind roller will be badly wound.

Due to a wide variation of the roller radius during the unwinding-winding process, the dynamic behavior of the system is considerably modified with time. To analyze this modification, let us consider the unwinder and winder separately. With quasi-static assumption on radius variations, the static gains between the control signals and web tensions appear to be proportional to the inverse of the radius [14]:

$$Gain_{DC}\left(\frac{T_u}{u_u}\right) = \frac{1}{R_u} \quad and \quad Gain_{DC}\left(\frac{T_w}{u_w}\right) = \frac{1}{R_w} \quad (8)$$

We therefore multiply the control signals by the corresponding radius measurement or estimation and controller synthesis is done using the plant which includes the radii multiplication (gain scheduling). This approach allows us to reduce web tension variations significantly despite velocity changes during processing [14, 12].

We synthesized a centralized H_∞ controller with output weighting and model matching (figure 3) for the system composed of equations (5), (6) and (7) without the vectors H_i. Model M_o gave the desired transfert function T_{yr}. In our case, M_o was a second order transfer function.

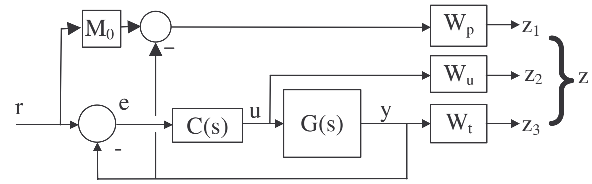

Fig. 3. S/KS/T weighting scheme with model matching for the design of H_∞ controllers

The weighting functions W_p, W_u, and W_t appear in the closed loop transfer matrix T_{zr} which is to be minimized. The weighting function W_p has a high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as follows [14]:

$$W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0} \quad (9)$$

where M is the maximum peak magnitude of the sensitivity S, ω_B is the required frequency bandwith ε_0 is the steady-state error allowed. The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies. The controller $C(s)$ is calculated using the "γ-iteration" [17]. To specify independently the tracking performance and robustness to perturbations, a two degree of freedom controller (for example a 2DOF H_∞ controller) can be used [11, 2].

To take into account in the control strategy the system inherent nonlinearities and some constant values (such as for example static frictions on rolls), model based feedforward signals have been added to the control signals. Thus, the control signals U_{si0}, which depend on the reference values of web tension and velocity and on the system state, are added to the H_∞ controller outputs u_i (Fig. 4). These feedforward signals are calculated online with the feedforward controller called C_{ff} in Fig. 4: U_{si0} cancels the H_i element where it appears (all elements of H_i can not be cancelled using input U_{si0}, see Appendix)

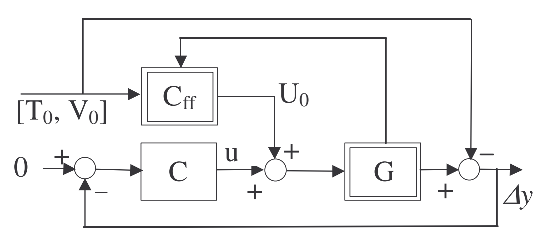

Fig. 4. Control strategy with feedforward signals

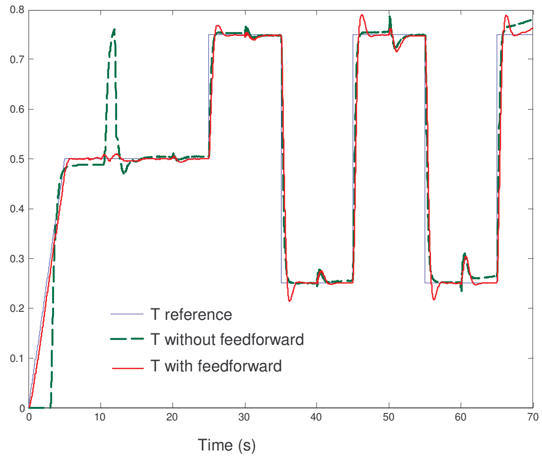

Fig. 5. Simulated web tensions (kg) for H_∞ centralized controller with and without additive feedforward signals

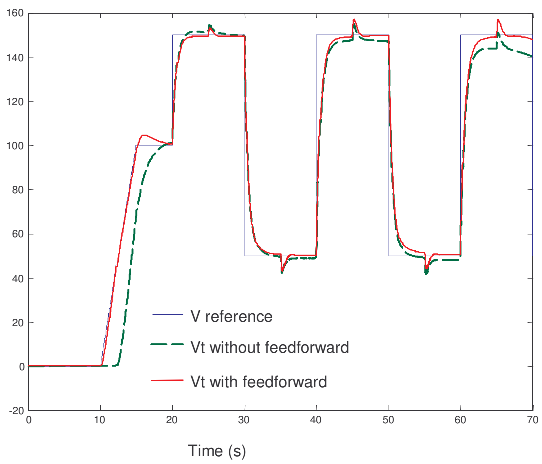

Fig. 6. Simulated web velocity (m/min) for H_∞ centralized controller with and without additive feedforward signals

Figure 5 and 6 present respectively the simulated web unwinder tension and web velocity (by assuming no sliping between web and master roll) for H_∞ controller with and without additive feedforward signals.
As expected, the centralized controller with additive feedforward signals improve not only the starting phase but also cancel the static errors to the web reference tension and velocity.

B. Semi-decentralized control design with or without overlapping

In industrial processes that contain a large number of actuators, it is inconvenient to use a global multivariable controller. An alternative solution is to use decentralized controllers, which allow reduction of the controller dimensions [12]. For example, in the case of a 9-motor plant, the system is decomposed into three subsystems: each subsystem contains three motors and is controlled independently by its local controller [5].

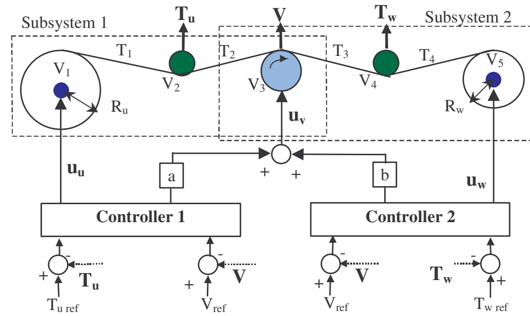

Fig. 7. Semi-decentralized overlapped control for the 3-motor plant

To reduce the coupling between two consecutive subsystems, an overlapping control strategy has been used. For instance, control signals of tractors located at the boundary of the two subsystems results from two controllers with output weighting *a* and *b* (Fig. 7). Such a decentralized overlapping control strategy has given good results in the case of a vehicle platoon [16].

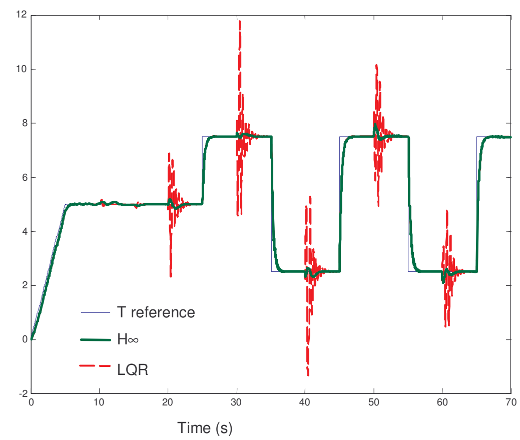

Fig. 8. Simulated web unwinding tensions (N) for H_∞ and *LQR* semi-decentralized controllers with overlapping and feedforward signals

Figure 8 shows simulation results for H_∞ and *LQR* semi-decentralized controllers with overlapping and feedforward additive signals discussed earlier. The results with the H_∞ overlapped controller is close to the centralized one whereas the *LQR* strategy leads to a strong velocity–tension coupling.

IV. Decentralized Control with Feedforward

The focus of this part is to design a completely decentralized controller for web processing lines. Therefore, the global system is divided into several subsystems with each subsystem containing only one actuator (see Fig. 13 in the Appendix).

A. Decentralized control design without subsystem decoupling

Each subsystem can be controlled by its own SISO H_∞ controller: one subsystem is under velocity control (master speed roll) whereas the other subsystems are under web tension control.

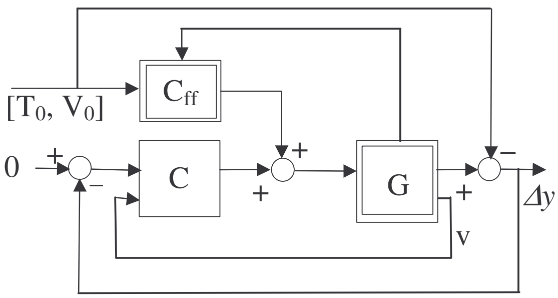

Fig. 9. Control strategy with additive measures and feedforward signals

To improve the dynamic behavior, additive measures can be included in the controller synthesis (Fig. 9); similar to the PI strategy presented in Fig. 2. The synthesis scheme is given in [26]. In our case, the web velocity measured or estimated in each subsystem is used as the second controller input.

Figure 10 shows the unwind tension simulation result for decentralized H_∞ controller (with feedforward signals) and with/without additive velocity measures. The two-input controller improves reference tracking while reducing velocity-tension coupling to some degree.

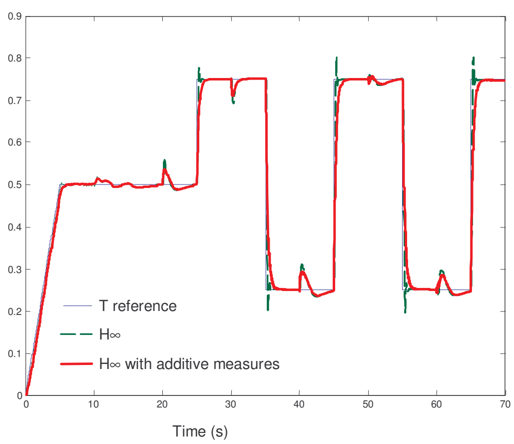

Fig. 10. Simulated unwinding web tension (kg) for completely decentralized controllers with feedforward signals: H_∞ with and without additive measures

B. *Decentralized control design with additional subsystem decoupling*

To facilitate web tension-velocity decoupling one requires improvement in decoupling individual subsystems. For example, the subsystem 2 affects subsystem 1 by the vectors $H_1 + A_{12}\underline{x}_2$: see Eq. (5). The sum of these vectors, called *P* in Fig. 11, can be calculated by assuming the state space vector is measured or estimated. It is therefore worthwhile to synthesize controllers that reject known perturbations [26].

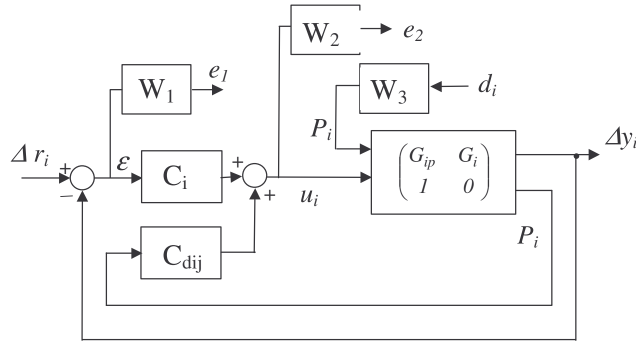

Fig. 11. H_∞ control design with additional subsystem decoupling

The synthesis scheme is represented on Fig. 11 where the weighting filter W_3 has been chosen as a constant. The implementation for a large scale system is presented on Fig. 13; in the figure C_{dij} represents the decoupling controller.

Preliminary results on a linear plant simulator show improvement in subsystem decoupling (see Fig. 12). The next step will be the selection of an adequate filter W_3 and its adjustment to adapt this strategy to the nonlinear process model.

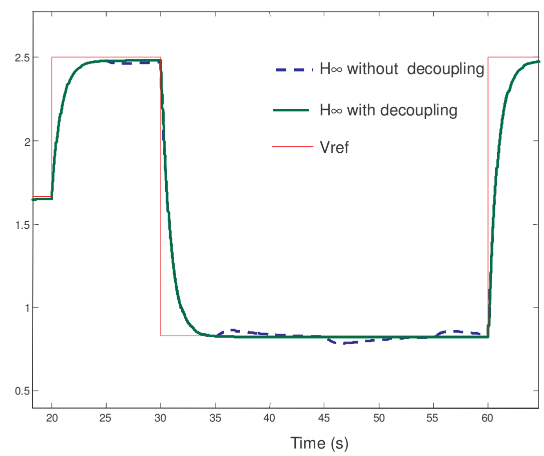

Fig. 12. Simulated web velocity (m/s) for decentralized controllers with feedforward signals: H_∞ with and without subsystems decoupling

V. Conclusion

Compared to decentralized PID controllers classically used in industrial web winding systems, multivariable H_∞ controllers had already shown improved web tension and speed decoupling. Web processing lines are generally of large scale and therefore it is not suitable to use a centralized controller for such processes. In this paper, a mathematically exact decentralized state space model for web processing lines is presented which leads to online calculation of feedforward control action. Different decentralized H_∞ control strategies with feedforward are then synthesized and validated on a realistic nonlinear web handling simulator. Future work will deal with fixed order controller synthesis using *BMI* optimization.

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Appendix

A1: Modeling of subsystem 1 and subsystem 2 :

Mechanical equation of the unwinder :

Nomenclature :

K_1	motor torque constant
U_{s1}	motor torque
C_{fsunw}	static friction coefficient
$f_{v1u}, f_{v1u}, f_{v1u}$	viscous friction coefficient
J_1	unwinder inertia
R_1	wound roll radius
l	web width
ρ	web mass density
h	web thickness

$$\frac{d}{dt}\left(J_1\Omega_1\right) = R_1T_1 - K_1U_{s1} - C_{fs_{unw}} - f_{v_{1u}}\Omega_1 + f_{v_{2u}}\Omega_1^2 - f_{v_{3u}}\Omega_1^3$$

with : $\frac{d}{dt}\left(J_1\Omega_1\right) = \frac{d}{dt}\left(J_1\frac{V_1}{R_1}\right) = \dot{J}_1\frac{V_1}{R_1} + J_1\frac{\dot{V}_1}{R_1} - J_1\frac{V_1\dot{R}_1}{R_1^2}$

and with the inertia calculation :

$$\begin{cases} J_1 = \frac{\pi}{2} l\,\rho\left(R_1^4 - R_{l_0}^4\right) \quad \Rightarrow \quad \dot{J}_1 = 2\pi\, l\,\rho\, R_1^3\,\dot{R}_1 \\ \dot{R}_1 \approx -\frac{h}{2\pi}\frac{V_1}{R_1} \end{cases}$$

the result is :

	$\frac{d}{dt}\left(J_1 \frac{V_1}{R_1}\right) = -2\pi\, l\, \rho R_1^3 \frac{h}{2\pi}\frac{V_1}{R_1}\frac{V_1}{R_1} + J_1 \frac{\dot{V}_1}{R_1} + J_1 \frac{V_1}{R_1^2}\frac{h}{2\pi}\frac{V_1}{R_1}$

with : $\begin{cases} V_1 = V_0 + v_1 \\ T_1 = T_0 + t_1 \\ U_{s1} = U_{s1_0} + u_{s1} \end{cases}$ the non-linear equation for small signals leads to :

$$J_1\dot{v}_1 = \left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right)V_0^2 - \frac{f_{v_{3u}}}{R_1^2}V_0^3 - R_1 C_{fs_{unw}} - f_{v_{1u}} V_0 + R_1^2 T_0 - R_1 K_1 U_{s1_0}\right)$$
$$+\left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right) + 3\frac{f_{v_{3u}}}{R_1^2}V_0\right)v_1^2 - \frac{f_{v_{3d}}}{R_1^2}v_1^3$$
$$+\left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right)2V_0 - 3\frac{f_{v_{3u}}}{R_1^2}V_0^2 - f_{v_{1u}}\right)v_1 + R_1^2 t_1 - R_1 K_1 u_{s1}$$

Mechanical equation of the unwinder tension sensor:

Nomenclature :	
f_2 viscous friction coefficient	$J_2\dot{V}_2 = -R_2^2(T_2 - T_1) - f_2 V_2$

with: $\begin{cases} V_2 = V_0 + v_2 \\ T_1 = T_0 + t_1 \\ T_2 = T_0 + t_2 \end{cases}$ the non-linear equation for small signals leads to : $J_2\dot{v}_2 = -f_2 V_0 - R_2^2 t_1 - f_2 v_2 + R_2^2 t_2$

Web span models:

Nomenclature :	
E Young modulus S web section Tb unwinding wound internal tension $E_0 = ES + T_0$	$L_1\dot{T}_1 = V_2(ES + T_1) - V_1(ES + 2T_1 - T_b)$ $L_2\dot{T}_2 = V_3(ES - T_2) - V_2(ES + 2T_2 - T_1)$

With $\begin{cases} V_1 = V_0 + v_1 \\ V_2 = V_0 + v_2 \\ T_1 = T_0 + t_1 \end{cases}$ and $\begin{cases} V_2 = V_0 + v_2 \\ V_3 = V_0 + v_3 \\ T_2 = T_0 + t_2 \end{cases}$ the non-linear equations for small signals lead to :

$$L_1\dot{t}_1 = V_0 T_b - V_0 T_0 - (E_0 + T_0 - T_b)v_1 - V_0 t_1 + E_0 v_2 + v_2 t_1 - 2 v_1 t_1$$
$$L_2\dot{t}_2 = V_0 t_1 - E_0 v_2 - 3V_0 t_2 + (E_0 - 2T_0)v_3 - 2V_0 T_0 + t_1 v_2 - 2 v_2 t_2 - v_3 t_2$$

State space representation of subsystem 1:

$E_1 \underline{\dot{x}}_1 = A_1 \underline{x}_1 + B_1 u_{s1} + H_1 + A_{12}\underline{x}_2$ with $\underline{x}_1^T = [v_1 \quad t_1 \quad v_2 \quad t_2]$

$$E_1 = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 \\ 0 & 0 & J_2 & 0 \\ 0 & 0 & 0 & L_2 \end{bmatrix} \quad A_1 = \begin{bmatrix} a_1 & R_1^2 & 0 & 0 \\ -(E_0 + T_0 - T_b) & -V_0 & -E_0 & 0 \\ 0 & -R_2^2 & -f_2 & R_2^2 \\ 0 & V_0 & -E_0 & -3V_0 \end{bmatrix} \quad B_1 = \begin{bmatrix} -K_1 R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E_0 - 2T_0 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \left(\rho S R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right)V_0^2 - \frac{f_{v_{3u}}}{R_1^2}V_0^3 - R_1 C_{fs_{unw}} - f_{v_{1u}}V_0 + R_1^2 T_0 - R_1 K_1 U_{s1_0} + \left(\rho S R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1} + 3\frac{f_{v_{3u}}}{R_1^2}V_0\right)v_1^2 - \frac{f_{v_{3u}}}{R_1^2}v_1^3 \\ V_0 T_b - V_0 T_0 + v_2 t_1 - 2 v_2 t_1 \\ -f_2 V_0 \\ -2V_0 T_0 + t_1 v_2 - 2 v_2 t_2 - v_3 t_2 \end{bmatrix}$$

where $a_1 = 2V_0\left(\rho S R_d^2 - \frac{J_d h}{2\pi R_d^2} + \frac{f_{v_{2d}}}{R_d}\right) - 3\frac{f_{v_{3d}}}{R_d^2}V_0^2 - f_{v_{1d}}$

State space representation of subsystem 2:

$$E_2 \dot{\underline{x}}_2 = A_2 \underline{x}_2 + B_2 u_{s2} + H_2 + A_{21}\underline{x}_1 + A_{23}\underline{x}_3 \quad \text{where} \quad \underline{x}_2^T = [v_3]$$

$$E_2 = [J_3] \quad A_2 = [a_2] \quad B_2 = [-K_2 R_2] \quad A_{21} = [0 \quad 0 \quad 0 \quad -R_3^2] \quad A_{23} = [R_3^2 \quad 0 \quad 0 \quad 0]$$

$$H_2 = \left[-R_3 C_{fs_{tra}} - f_{v_{1t}} V_0 + \frac{f_{v_{2t}}}{R_3} V_0^2 - \frac{f_{v_{3t}}}{R_3^2} V_0^3 + R_3 K_3 U_{s2_0} + R_3^{\,2}\left(T_{e_0} - T_{d_0}\right) + \left(\frac{f_{v_{2t}}}{R_3} - 3\frac{f_{v_{3t}}}{R_3^2} V_0\right) v_3^2 - \frac{f_{v_{3t}}}{R_3^2} v_3^3\right]$$

where $a_2 = -f_{v_{1t}} + \frac{f_{v_{2t}}}{R_t} 2V_0 - 3\frac{f_{v_{3t}}}{R_t^2} V_0^2$

A2 : Decentralized control strategy for a large scale web transport system :

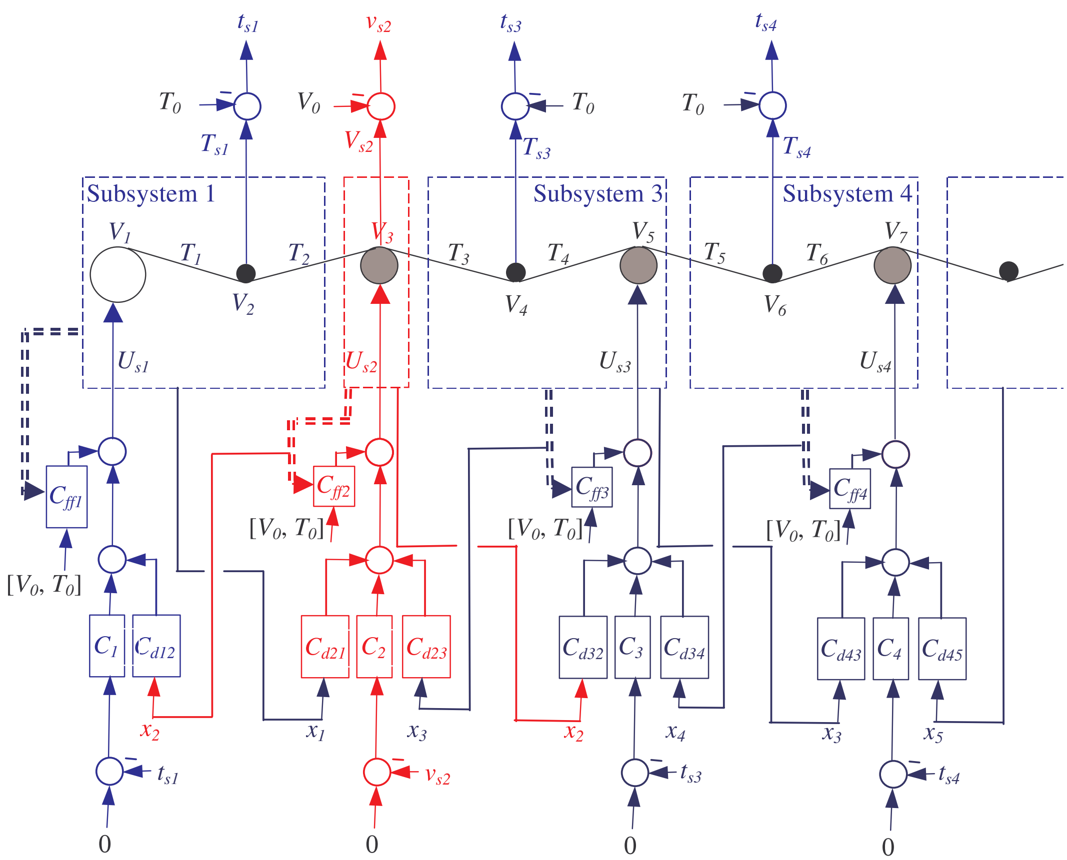

Fig. 13. Decentralized control strategy for a large scale web transport system