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Analysis and Control of a Class of Large-Scale Interconnected Nonlinear Systems

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Abstract. In this paper, decentralized controller design and stability analysis of a class of large-scale interconnected nonlinear systems is presented. Each subsystem in the overall system is nonlinear and so are the interconnections that join the subsystems. For controller synthesis the interconnecting nonlinear functions are known to satisfy a bound which is a polynomial in the states. The nonlinear interconnections need not satisfy any symmetry property. We show global exponential stability of the closed-loop interconnected system for the proposed decentralized controller. Robustness of the decentralized controller to a class of interconnections that do not satisfy matching conditions is also shown.

1 Introduction

Large-scale interconnected systems appear in a variety of engineering applications such as power systems, large structures, manufacturing processes such as web handling systems. Decentralized control of large-scale interconnected systems has been a topic of interest for several decades. Decentralized and robust control of large-scale uncertain systems has been investigated actively by several researchers. A large body of research in this area has been reported in [1]. [2] consider decentralized control for uncertain systems and show uniform ultimate boundedness of the state in the presence of higher order uncertainties. Decentralized adaptive control of interconnected systems can be found in [3, 4]. A nice presentation of string stability of interconnected systems with applications to vehicle following can be found in [5].

In this work we present decentralized controller design and stability analysis for large-scale interconnected nonlinear systems. No particular structure for the nonlinearities within a subsystem (i.e., depending on the state of the subsystem) or the interconnecting nonlinear functions has been assumed. In this paper, we assume that each subsystem is directly connected to just the neighboring subsystems. This assumption can be easily removed by a simple modification of the existing controller, and we do not present this case due to lack of space. The uniqueness of this work stems from the fact that global exponential stability has been shown for a decentralized control scheme with higher order nonlinear interconnections.

The rest of the paper is organized as follows. Controller design and stability has been considered in section 2. In section 2, we first consider a simple case where the interconnected system is composed of N first order nonlinear systems. Then we extend controller design and show stability for a general case of each subsystem of higher order. In each case we show global exponential stability for the proposed decentralized controller. Robustness of the decentralized controller to a class of nonlinear interconnections that do not satisfy the matching conditions is also shown in section 2. Results of the present work are summarized and some important issues that can be considered for future work are given in section 3.

2 Controller Design and Stability

The class of large-scale interconnected systems considered are of the form:

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{f}_i(\mathbf{x}_i) + \mathbf{g}_i(\mathbf{x}_i)u_i \\ &\quad + \mathbf{f}_{i,i-1}(\mathbf{x}_i, \mathbf{x}_{i-1}) + \mathbf{f}_{i,i+1}(\mathbf{x}_i, \mathbf{x}_{i+1}) \\ y_i &= h_i(\mathbf{x}_i) \end{aligned} \tag{1}$$

where $i = 1, \dots, N$, $\mathbf{x}_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ denote the state, input and output of the i-th subsystem. We assume that the functions $\mathbf{f}_i(\mathbf{x}_i)$, $\mathbf{g}_i(\mathbf{x}_i)$, $\mathbf{f}_{i,i-1}(\mathbf{x}_i, \mathbf{x}_{i-1})$ and $\mathbf{f}_{i,i+1}(\mathbf{x}_i, \mathbf{x}_{i+1})$ are smooth vectors in $\mathbb{R}^{n_i}$. $\mathbf{f}_i(\mathbf{x}_i)$ denotes the nonlinearities within the i-th subsystem. The interconnecting nonlinearities $\mathbf{f}_{i,i-1}(\mathbf{x}_i, \mathbf{x}_{i-1})$ and $\mathbf{f}_{i,i+1}(\mathbf{x}_i, \mathbf{x}_{i+1})$ are assumed to be dependent only on the neighboring subsystems. Figure 1 below depicts such a scenario. The interconnections are as-

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Figure 1: Large-scale interconnected system

sumed to be polynomially bounded in states. The goal of this paper is to develop a decentralized control algorithm for each subsystem so that the overall interconnected system is globally exponentially stable.

2.1 Simple Case

Before considering the generalized system, we consider the case of a large-scale interconnected system composed of N first-order subsystems given by

$$\begin{aligned}\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ &\quad + f_{i,i-1}(x_i, x_{i-1}) + f_{i,i+1}(x_i, x_{i+1}) \\ y_i &= x_i\end{aligned} \tag{2}$$

where $i = 1, \ldots, N$, $g_i(x_i) \neq 0$, $x_i, u_i, y_i \in \mathbb{R}$ denote the state, input, and the output, respectively. The following lemma gives the control law, $u_i(t)$, for each subsystem and the conditions under which the interconnected system (2) is globally exponentially stable. Also, in the remainder of the paper all greek letter constants introduced will be positive constants.

Lemma 2.1 *Suppose the following conditions are satisfied for the interconnected system of* (2).

(1) There exists a feedback control law, $u_i = -(1/g(x_i))\alpha_i(x_i)$, that renders the interconnection-free subsystem $\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$ globally exponentially stable. Hence by converse Lyapunov theorem there exists a Lyapunov function, $V_i(x_i)$, and positive constants $\beta_{1,i}, \beta_{2,i}, \beta_{3,i}$, and $\beta_{4,i}$, such that

$$\beta_{1,i}|x_i|^2 \leq V_i(x_i) \leq \beta_{2,i}|x_i|^2$$
$$\frac{\partial V_i}{\partial x_i}[f_i(x_i) - \alpha_i(x_i)] \leq -\beta_{3,i}|x_i|^2$$
$$\left|\frac{\partial V_i}{\partial x_i}\right| \leq \beta_{4,i}|x_i|$$

(2) There exist constants $\beta_{5,i}$ and $\beta_{6,i}$ such that

$$|f_{i,i-1}(x_i, x_{i-1})| \leq \beta_{5,i}|x_i|^{p_{1,i}}|x_{i-1}|^{q_{1,i}}$$
$$|f_{i,i+1}(x_i, x_{i+1})| \leq \beta_{6,i}|x_i|^{p_{2,i}}|x_{i+1}|^{q_{2,i}}$$

where $p_{1,i}, q_{1,i}, p_{2,i}$ and $q_{2,i}$ are some known positive integers.

Under the above conditions there exists a feedback law of the form

$$u_i = -\left(\frac{1}{g_i(x_i)}\right)\{\alpha_i(x_i) + \alpha_{ii}(x_i)\}$$

that guarantees global exponential stability of the interconnected system given by (2).

Proof: Consider the following for the second part, $\alpha_{ii}(x_i)$, of the control input

$$\begin{aligned}\alpha_{ii}(x_i) = &-\delta_{1,i}\frac{\partial V_i}{\partial x_i}|x_i|^{2p_{1,i}} - \delta_{2,i}\frac{\partial V_i}{\partial x_i}|x_i|^{2q_{1,i}-2} \\ &- \delta_{3,i}\frac{\partial V_i}{\partial x_i}|x_i|^{2p_{2,i}} - \delta_{4,i}\frac{\partial V_i}{\partial x_i}|x_i|^{2q_{2,i}-2}\end{aligned} \tag{3}$$

where $\delta_{k,i} : k = 1, \ldots, 4, i = 1, \ldots, N$, are positive constants. Also, $\delta_{1,1} = \delta_{4,1} = \delta_{2,N} = \delta_{3,N} \equiv 0$. The control input requires the knowledge of a Lyapunov function for the i-th subsystem. One such Lyapunov function for this system is clearly $V_i(x_i) = (1/2)x_i^2$. The derivative of the Lyapunov function gives

$$\begin{aligned}\dot{V}_i &= \frac{\partial V_i}{\partial x_i}\dot{x}_i \\ &= \frac{\partial V_i}{\partial x_i}[f_i(x_i) - \alpha_i(x_i)] - \frac{\partial V_i}{\partial x_i}\alpha_{ii}(x_i) \\ &\quad + \frac{\partial V_i}{\partial x_i}\{f_{i,i-1}(x_i, x_{i-1}) + f_{i,i+1}(x_i, x_{i+1})\}\end{aligned}$$

Using conditions (1) and (2), we obtain,

$$\begin{aligned}\dot{V}_i \leq &-\beta_{3,i}|x_i|^2 - \frac{\partial V_i}{\partial x_i}\alpha_{ii}(y_i) \\ &+ \beta_{4,i}\beta_{5,i}|x_i|^{p_{1,i}+1}|x_{i-1}|^{q_{1,i}} \\ &+ \beta_{4,i}\beta_{6,i}|x_i|^{p_{2,i}+1}|x_{i+1}|^{q_{2,i}}\end{aligned}$$

Now, consider the following Lyapunov function for the interconnected system (2),

$$V(x_1, \ldots, x_N) = \sum_{i=1}^{N} V_i(x_i)$$

Substituting $\alpha_{ii}(x_i)$ from (3) into the time derivative of V gives,

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{N} \{&-\beta_{3,i}\|x_i\|^2 - \delta_{1,i}|x_i|^{2p_{1,i}+2} - \delta_{2,i}|x_i|^{2q_{1,i}} \\ &- \delta_{3,i}|x_i|^{2p_{2,i}+2} - \delta_{4,i}|x_i|^{2q_{2,i}} \\ &+ \beta_{4,i}\beta_{5,i}|x_i|^{p_{1,i}+1}|x_{i-1}|^{q_{1,i}} \\ &+ \beta_{4,i}\beta_{6,i}|x_i|^{p_{2,i}+1}|x_{i+1}|^{q_{2,i}} \ \}\end{aligned}$$

In the above expression, when summation is evaluated we take $x_0 = 0$ and $x_{N+1} = 0$. Rearranging terms we obtain

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N} \{\beta_{3,i}|x_i|^2\} \\ & -\sum_{k=2}^{N} \{ \, \delta_{1k}|x_k|^{2p_{1,k}+2} + \delta_{2k-1}|x_{k-1}|^{2q_{1,k-1}} \\ & \quad - \beta_{4,k}\beta_{5,k}|x_k|^{p_{1,k}+1}|x_{k-1}|^{q_{1,k-1}} \, \} \\ & -\sum_{k=2}^{N} \{ \, \delta_{3,k-1}|x_{k-1}|^{2p_{2,k-1}+2} + \delta_{4,k}|x_k|^{2q_{2,k}} \\ & \quad - \beta_{4,k-1}\beta_{6,k-1}|x_{k-1}|^{p_{2,k-1}+1}|x_{k-1}|^{q_{2,k}} \, \}\end{aligned}$$

Choosing $\delta_{1,k}, \delta_{2,k-1}, \delta_{3,k-1}$ and $\delta_{4,k}$, $k = 2 \dots N$, such that

$$\begin{aligned}2\sqrt{\delta_{1,k}}\sqrt{\delta_{2,k-1}} &\geq \beta_{4,k}\beta_{5,k} \\ 2\sqrt{\delta_{3,k-1}}\sqrt{\delta_{4,k}} &\geq \beta_{4,k-1}\beta_{6,k-1}\end{aligned}$$

we obtain the following:

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N} \{\beta_{3,i}|x_i|^2\} \\ & -\sum_{k=2}^{N} \left\{ \sqrt{\delta_{1,k}}|x_k|^{p_{1,k}+1} - \sqrt{\delta_{2,k-1}}|x_{k-1}|^{2q_{1,k-1}} \right\}^2 \\ & -\sum_{k=2}^{N} \left\{ \sqrt{\delta_{3,k-1}}|x_{k-1}|^{p_{2,k-1}+1} - \sqrt{\delta_{4,k}}|x_k|^{q_{2,k}} \right\}\end{aligned}$$

Thus,

$$\dot{V} \leq -\sum_{i=1}^{N} \beta_{3,i}|x_i|^2$$

Taking $\mathbf{x}^T = [x_1, x_2, \dots, x_N]^T$ and $\beta_{3min} = \min_i \beta_{3,i}$, we get

$$\dot{V}(t, \mathbf{x}) \leq -2\beta_{3min} V(t, \mathbf{x})$$

which implies

$$V(t, \mathbf{x}) \leq V(t_0, \mathbf{x}(t_0)e^{-2\beta_{3min}(t-t_0)}$$

Thus, the interconnected system is globally exponentially stable, $\mathbf{x}(t) \to 0$ exponentially.

■

Remark 2.1 *The bounds on $f_{i,i-1}(x_i, x_{i-1})$ and $f_{i,i+1}(x_i, x_{i+1})$ in condition (2) of lemma (2.1) are assumed to be in the product form. It should be noted that bounds of the following type*

$$\begin{aligned}|f_{i,i-1}(x_i, x_{i-1})| &\leq \beta_{51,i}|x_i|^{p_{3,i}} + \beta_{52,i}|x_{i-1}|^{q_{3,i}} \\ |f_{i,i+1}(x_i, x_{i+1})| &\leq \beta_{61,i}|x_i|^{p_{4,i}} + \beta_{62,i}|x_{i+1}|^{q_{4,i}}\end{aligned}$$

can also be handled by a simple modification of $\alpha_{ii}(x_i)$ given by (3). In fact, the inequality $2|a||b| \leq |a|^2 + |b|^2$, shows similar nature of the two types of bounds in question.

So far we have assumed that the interconnecting nonlinearities, $f_{i,i-1}(x_i, x_{i-1})$ and $f_{i,i+1}(x_i, x_{i+1})$, are bounded by a single exponent of the state. The following corollary considers the bounds on $f_{i,i-1}(x_i, x_{i-1})$ and $f_{i,i+1}(x_i, x_{i+1})$ to be polynomials.

Corollary 2.1 *Suppose the bounds on $f_{i,i-1}(x_i, x_{i-1})$ and $f_{i,i+1}(x_i, x_{i+1})$ are given by*

$$\begin{aligned}|f_{i,i-1}(x_i, x_{i-1})| &\leq \sum_{j=1}^{p_{1,i}}\sum_{k=1}^{q_{1,i}} \beta_{j,k}^{4,i}|x_i|^j|x_{i-1}|^k \\ |f_{i,i+1}(x_i, x_{i+1})| &\leq \sum_{j=1}^{p_{2,i}}\sum_{k=1}^{q_{2,i}} \beta_{j,k}^{5,i}|x_i|^j|x_{i+1}|^k\end{aligned}$$

Then the following control input guarantees global exponential stability of the interconnected nonlinear system given by (2).

$$\begin{aligned}\alpha_{ii}(x_i) = -\frac{\partial V_i}{\partial x_i} \{ & \sum_{k=1}^{q_{1,i}}\sum_{j=1}^{p_{1,i}} \delta_{1,i}^{j,k}|x_i|^{2j} + \sum_{j=1}^{p_{1,i}}\sum_{k=1}^{q_{1,i}} \delta_{2,i}^{j,k}|x_i|^{2k-2} \\ & + \sum_{k=1}^{q_{2,i}}\sum_{j=1}^{p_{2,i}} \delta_{3,i}^{j,k}|x_i|^{2j} + \sum_{j=1}^{p_{2,i}}\sum_{k=1}^{q_{2,i}} \delta_{4,i}^{j,k}|x_i|^{2k-2}\end{aligned}$$

where $\delta_{1,i}^{j,k}, \delta_{2,i}^{j,k}, \delta_{3,i}^{j,k}$ and $\delta_{4,i}^{j,k}$ are chosen to satisfy the following,

$$\begin{aligned}2\sqrt{\delta_{1,i}^{j,k}}\sqrt{\delta_{2,i-1}^{j,k}} &\geq \beta_{4,i}^{j,k}\beta_{5,i}^{j,k} \\ 2\sqrt{\delta_{3,i-1}^{j,k}}\sqrt{\delta_{4,i}^{j,k}} &\geq \beta_{4,i}^{j,k}\beta_{5,i}^{j,k}\end{aligned}$$

Proof: The proof of this corollary follows a similar procedure as the proof of lemma (2.1). ■

2.2 General Case

Now we consider the general system as given by (1). Consider the interconnection-free i-th subsystem,

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{f}_i(\mathbf{x}_i) + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i &= h_i(\mathbf{x}_i)\end{aligned}$$

where $\mathbf{x}_i \in \mathbb{R}^{n_i}$. We assume that the interconnection-free i-th subsystem is globally input-output linearizable with relative degree n_i, which means that $\forall \mathbf{x}_i \in \mathbb{R}^{n_i}$ the following are true[6]:

1. $L_{\mathbf{g}_i}L_{\mathbf{f}_i}^k h_i(\mathbf{x}_i) = 0$, for $k = 0, \dots, n_i - 2$;

2. $s_i(\mathbf{x}) = L_{\mathbf{g}_i}L_{\mathbf{f}_i}^{n_i-1}h_i(\mathbf{x}_i)$ is nonsingular.

Define $r_i(\mathbf{x}) = L_{\mathbf{f}_i}h_i(\mathbf{x})$. If we apply a control input u_i such that

$$v_i = q_i(x_i) + s_i(x_i)u_i$$

then the closed-loop interconnected system, (1), is described by

$$\begin{aligned}\dot{\mathbf{z}}_i &= \mathbf{A}_i\mathbf{z}_i + \mathbf{B}_iv_i \\ &\quad + \mathbf{f}_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1}) + \mathbf{f}_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1})\end{aligned} \tag{4}$$

where $(\mathbf{A}_i, \mathbf{B}_i)$ is in Brunovsky canonical form. We also assume that the interconnecting nonlinearities satisfy the matching condition, i.e.,

$$\mathbf{f}_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1}) = \mathbf{B}_iw_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1}) \tag{5}$$

$$\mathbf{f}_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1}) = \mathbf{B}_iw_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1}) \tag{6}$$

where $w_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1})$ and $w_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1})$ are polynomially bounded as follows:

$$||w_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1})|| \leq \sum_{j=1}^{p_{1,i}}\sum_{k=1}^{q_{1,i}} \beta_{j,k}^{4,i}||\mathbf{z}_i||^j||\mathbf{z}_{i-1}||^k$$

$$||w_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1})|| \leq \sum_{j=1}^{p_{2,i}}\sum_{k=1}^{q_{2,i}} \beta_{j,k}^{5,i}||\mathbf{z}_i||^j||\mathbf{z}_{i+1}||^k$$

We consider the following control input for $v_i(t)$,

$$\begin{aligned}v_i(t) = -\mathbf{K}_i^T\mathbf{z}_i - \operatorname{sgn}(\mathbf{z}_{in_i})\{ \\ \sum_{k=1}^{q_{1,i}}\sum_{j=1}^{p_{1,i}}\left(\delta_{1,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{2,i}^{j,k}||\mathbf{z}_i||^{2k}\right) \\ + \sum_{k=1}^{q_{2,i}}\sum_{j=1}^{p_{2,i}}\left(\delta_{3,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{4,i}^{j,k}||\mathbf{z}_i||^{2k}\right)\}\end{aligned} \tag{7}$$

where $\delta_{1,i}^{j,k}, \delta_{2,i}^{j,k}, \delta_{3,i}^{j,k}$ and $\delta_{4,i}^{j,k}$ are chosen to satisfy the following,

$$2\sqrt{\delta_{1,i}^{j,k}}\sqrt{\delta_{2,i-1}^{j,k}} \geq \beta_{4,i}^{j,k}\beta_{5,i}^{j,k} \tag{8}$$

$$2\sqrt{\delta_{3,i-1}^{j,k}}\sqrt{\delta_{4,i}^{j,k}} \geq \beta_{4,i}^{j,k}\beta_{5,i}^{j,k} \tag{9}$$

Now consider the following Lyapunov function candidate,

$$V_i(\mathbf{z}_i) = \frac{1}{2}\mathbf{z}_i^T\mathbf{z}_i$$

Taking the time derivative and simplifying we obtain

$$\begin{aligned}\dot{V}_i &= \mathbf{z}_i^T(\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_i^T)\mathbf{z}_i \\ &\quad - \mathbf{z}_i^T\mathbf{B}_i\operatorname{sgn}(\mathbf{z}_{in_i})\{ \\ &\quad \sum_{k=1}^{q_{1,i}}\sum_{j=1}^{p_{1,i}}\left(\delta_{1,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{2,i}^{j,k}||\mathbf{z}_i||^{2k}\right) \\ &\quad + \sum_{k=1}^{q_{2,i}}\sum_{j=1}^{p_{2,i}}\left(\delta_{3,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{4,i}^{j,k}||\mathbf{z}_i||^{2k}\right)\} \\ &\quad + \mathbf{z}_i^T\mathbf{B}_i(w_{i,i-1}(\mathbf{z}_i, \mathbf{z}_{i-1}) + w_{i,i+1}(\mathbf{z}_i, \mathbf{z}_{i+1}))\end{aligned}$$

Notice that

$$\mathbf{z}_i^T\mathbf{B}_i\operatorname{sgn}(z_{in_i}) = |z_{in_i}|$$

where z_{in_i} denotes the n_i component of the state vector $\mathbf{z}_i$. Since $(\mathbf{A}_i, \mathbf{B}_i)$ is in Brunovsky canonical form we get

$$\mathbf{z}_i^T\mathbf{B}_i = z_{in_i}, \implies |\mathbf{z}_i^T\mathbf{B}_i| = |z_{in_i}|$$

Also since $(\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_i^T)$ has all its eigenvalues strictly negative by choice of feedback gain vector $\mathbf{K}_i$, let λ_i^{min} be the magnitude of the largest eigenvalue of $(\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_i^T)$. Then

$$\begin{aligned}\dot{V}_i &\leq -\lambda_i^{min}\mathbf{z}_i^T\mathbf{z}_i - |z_{in_i}|\{ \\ &\quad \sum_{k=1}^{q_{1,i}}\sum_{j=1}^{p_{1,i}}\left(\delta_{1,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{2,i}^{j,k}||\mathbf{z}_i||^{2k}\right) \\ &\quad + \sum_{k=1}^{q_{2,i}}\sum_{j=1}^{p_{2,i}}\left(\delta_{3,i}^{j,k}||\mathbf{z}_i||^{2j} + \delta_{4,i}^{j,k}||\mathbf{z}_i||^{2k}\right)\} \\ &\quad + |z_{in_i}|\sum_{j=1}^{p_{1,i}}\sum_{k=1}^{q_{1,i}}\beta_{j,k}^{4,i}||\mathbf{z}_i||^j||\mathbf{z}_{i-1}||^k \\ &\quad + |z_{in_i}|\sum_{j=1}^{p_{2,i}}\sum_{k=1}^{q_{2,i}}\beta_{j,k}^{5,i}||\mathbf{z}_i||^j||\mathbf{z}_{i+1}||^k\end{aligned}$$

Now consider the following Lyapunov function candidate for the interconnected system given by (4),

$$V(\mathbf{z}_1, \ldots, \mathbf{z}_N) = \sum_{i=1}^{N} V_i(\mathbf{z}_i)$$

Choose $\delta_{1,i}^{j,k}, \delta_{2,i}^{j,k}, \delta_{3,i}^{j,k}$ and $\delta_{4,i}^{j,k}$ such that (8) and (9) are satisfied. Taking the time derivative of the overall Lyapunov function we get,

$$\begin{aligned}\dot{V} &\leq -\sum_{i=1}^{N}\lambda_i^{min}\mathbf{z}_i^T\mathbf{z}_i - \sum_{i=2}^{N}|\mathbf{z}_{in_i}|\{ \\ &\quad \sum_{k=1}^{q_{1,i}}\sum_{j=1}^{p_{1,i}}\left(\sqrt{\delta_{1,i}^{j,k}}||\mathbf{z}_i||^j - \sqrt{\delta_{2,i-1}^{j,k}}||\mathbf{z}_i||^k\right)^2 \\ &\quad + \sum_{k=1}^{q_{2,i}}\sum_{j=1}^{p_{2,i}}\left(\sqrt{\delta_{3,i-1}^{j,k}}||\mathbf{z}_i||^j - \sqrt{\delta_{4,i}^{j,k}}||\mathbf{z}_i||^k\right)^2\}\end{aligned}$$

Thus,

$$\dot{V} \leq -\sum_{i=1}^{N}\lambda_i^{min}\mathbf{z}_i^T\mathbf{z}_i \tag{10}$$

Let $\lambda = \min_i \lambda_i^{min}$, and $\mathbf{z}^T = [\mathbf{z}_1^T, \mathbf{z}_2^T, \ldots, \mathbf{z}_N^T]$, then

$$\dot{V}(t, \mathbf{z}) \leq -2\lambda V(t, \mathbf{z})$$

which implies

$$V(t, \mathbf{z}) = V(t_0, \mathbf{z}(t_0))e^{-2\lambda(t-t_0)}$$

Thus, the interconnected system (4) is globally exponentially stable.

2.3 Robustness to nonlinear interconnections not satisfying matching conditions

Suppose after input-output feedback linearization, the interconnecting nonlinear functions do not satisfy the matching conditions given by (5) and (6). Consider the following,

$$\begin{aligned}\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1}) &= \mathbf{B}_i w_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1}) \\ &\quad + \Delta\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1}) \\ \mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1}) &= \mathbf{B}_i w_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1}) \\ &\quad + \Delta\mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1})\end{aligned}$$

where $\Delta\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1}) \in \mathbb{R}^{n_i}$ and $\Delta\mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1}) \in \mathbb{R}^{n_i}$ denote the unmatched part of $\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1})$ and $\mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1})$, respectively.

Lemma 2.2 *The control law given by* (7) *is robust to all uncertainties* $\Delta\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1})$ *and* $\Delta\mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1})$ *which are bounded by*

$$\begin{aligned}||\Delta\mathbf{f}_{i,i-1}(\mathbf{z}_i,\mathbf{z}_{i-1})|| &\le \eta_{1,i}||\mathbf{z}_i|| + \eta_{2,i}||\mathbf{z}_{i-1}|| \\ ||\Delta\mathbf{f}_{i,i+1}(\mathbf{z}_i,\mathbf{z}_{i+1})|| &\le \eta_{3,i}||\mathbf{z}_i|| + \eta_{4,i}||\mathbf{z}_{i+1}||\end{aligned}$$

Proof: Due to these unmatched parts the derivative of the overall Lyapunov function given by (10) is modified to,

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N} &-\lambda_i^{min}\mathbf{z}_i^T\mathbf{z}_i + (\eta_{1,i}+\eta_{4,i})||\mathbf{z}_i||^2 \\ &+ \eta_{2,i}||\mathbf{z}_i||||\mathbf{z}_{i-1}|| + \eta_{3,i}||\mathbf{z}_i||||\mathbf{z}_{i+1}||\end{aligned}$$

We assume that λ_i^{min} is chosen large enough by choosing the feedback gain vector $\mathbf{K}_i^T$ such that $\lambda_i^{min} = \kappa_i^{min} + \eta_{1,i} + \eta_{4,i} + \nu_i$, then

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N} \{ &- \kappa_i^{min}\mathbf{z}_i^T\mathbf{z}_i - \nu_i||\mathbf{z}_i||^2 \\ &+ \eta_{2,i}||\mathbf{z}_i||||\mathbf{z}_{i-1}|| + \eta_{3,i}||\mathbf{z}_i||||\mathbf{z}_{i+1}|| \ \}\end{aligned}$$

Expanding the summation we obtain,

$$\begin{aligned}\dot{V} &\le -\sum_{i=1}^{N}\kappa_i^{min}\mathbf{z}_i^T\mathbf{z}_i \\ &\quad - \{\nu_1||\mathbf{z}_1||||^2 + \cdots + \nu_N||\mathbf{z}_N||||^2\} \\ &\quad + \{\eta_{2,2}||\mathbf{z}_2||||\mathbf{z}_1|| + \cdots + \eta_{2,N}||\mathbf{z}_N||||\mathbf{z}_{N-1}||\} \\ &\quad + \{\eta_{3,1}||\mathbf{z}_1||||\mathbf{z}_2|| + \cdots + \eta_{3,N-1}||\mathbf{z}_{N-1}||||\mathbf{z}_N||\} \\ &\le -\sum_{i=1}^{N}\kappa_i^{min}\mathbf{z}_i^T\mathbf{z}_i \\ &\quad - \{\nu_1||\mathbf{z}_1||^2 + \cdots + \nu_N||\mathbf{z}_N||||^2\} \\ &\quad + \{\ (\eta_{2,2}+\eta_{3,1})||\mathbf{z}_1||||\mathbf{z}_2|| + \\ &\qquad \cdots + (\eta_{2,N}+\eta_{3,N-1})||\mathbf{z}_{N-1}||||\mathbf{z}_N||\ \}\end{aligned}$$

Choose ν_i such that

$$2\sqrt{\nu_i}\sqrt{\nu_{i+1}} \ge (\eta_{2,i+1}+\eta_{3,i}); \quad i = 1,\ldots,N-1$$

Using the above choice of ν_i's we can complete squares to obtain,

$$\dot{V} \le -\sum_{i=1}^{N}\kappa_i^{min}\mathbf{z}_i^T\mathbf{z}_i$$

■

3 Discussion

In the present work, we considered decentralized control design and stability of a class of large-scale interconnected systems. The decentralized controller has been shown to provide global exponential stability of the overall interconnected system. We have also shown that the proposed decentralized controller is robust to a class of interconnections that do not satisfy matching conditions. One issue that has important "real-world" applications is the performance of such controllers in the presence of external disturbances. Another issue that is equally important is to attenuate disturbances that appear in individual subsystems and to minimize disturbance propagation to other subsystems. These issue will be explored in future work.

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