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Robust Decentralized Control of Large-Scale Interconnected Systems: General Interconnections

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Abstract In this paper, a new decentralized control scheme is developed for a large-scale interconnected nonlinear systems with uncertain but bounded nonlinear interconnections. The interconnections are assumed to be bounded by polynomial type nonlinearities in states. If the interconnections are bounded by a pth-order polynomial in states, then the proposed controller has terms involving pth-order or less. This is in sharp contrast to the existing literature, which use a $(2p-1)$th-order terms in the controller. We develop robust designs if the coefficients of the bounded polynomial are known, and adaptive designs if the coefficients are not known. We show global exponential convergence of the states for the robust case and global asymptotic convergence of the states for the adaptive case. First, we consider systems that satisfy matching conditions and then extend the designs for systems that do not satisfy matching conditions. We give several examples to illustrate the design methodology. Further, we show how our designs can be extended to interconnections that cannot be bounded by finite length polynomials.

1 Introduction

Large-scale interconnected systems appear in a variety of engineering applications such as power systems, large structures, and manufacturing processes such as web handling systems. Decentralized control of large-scale interconnected systems has been a topic of interest for several decades. Decentralized control schemes present a practical and efficient means for designing control algorithms that utilize just the state of each subsystem without any information from other subsystems. A large body of research in this area has been reported in [1]. Recent research in this area can be found in [2, 3, 4]. [2] considers decentralized control for uncertain systems and show uniform ultimate boundedness of the state in the presence of higher-order uncertainties. String stability of interconnected systems with applications to vehicle following can be found in [4]. Decentralized adaptive control of interconnected systems that are in strict feedback form can be found in [5, 3]. Robustness is a primary concern in all large-scale interconnected systems. Areas of attention is often addressed towards uncertain interconnections and stability of such uncertain systems. Extensive literature in the area of uncertain systems can be found in the survey paper [6].

In the previous work [2, 3], to compensate a nonlinear interconnection bounded by a pth-order polynomial in states, decentralized controllers utilize a $(2p-1)$th-order terms in states to render the closed-loop system stable. The motivation behind this work comes from asking the question: why does a decentralized controller require $(2p-1)$th-order terms to compensate for pth order interconnections? The higher order controller appears because of the controller is designed via Lyapunov method, that uses a quadratic function in states as the Lyapunov function candidate. In this work, we use max function in states as our Lyapunov function candidate to synthesize stable decentralized controllers that involve pth-order terms in states. The resulting decentralized controller is discontinuous. The proposed controller scheme works for systems with both matched and unmatched uncertainties. Throughout the paper we present simulation results for the proposed designs.

The rest of the paper is organized as follows. In Section 1, we consider the large-scale interconnected system that satisfies matching conditions. In Section 2, we develop the decentralized controller for an example and compare it with other existing decentralized controllers. In Section 3, we extend the control technique to general system given in Section 1. Extension to adaptive designs is shown in Section 4. In Section 5, we design decentralized controllers for large-scale systems in strict feedback form. In Section 6, we show how the proposed technique can be extended for interconnections that cannot be bounded by finite length polynomials, but by known functions in states. Conclusions are given in Section 7.

2 Large-scale Nonlinear Systems

Consider the large-scale interconnected system

$$\dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{g}_i(\mathbf{x}_i)\tau_i + \mathbf{f}_i^i(\mathbf{x}_1, \dots, \mathbf{x}_N) \quad (1)$$

where $i = 1, \dots, N$ denotes each subsystem, $\mathbf{x}_i \in \mathbb{R}^{n_i}$ and $\tau_i \in \mathbb{R}$ denote the state and input, respectively, of the i-th subsystem. The vector fields

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$\mathbf{f}_i(\mathbf{x}_i)$, $\mathbf{g}_i(\mathbf{x}_i)$, $\mathbf{f}_i^i(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$ are assumed to be smooth with $\mathbf{f}_i(0) = 0$, $\mathbf{g}_i(0) \neq 0$, $\mathbf{f}_i^i(0, 0, \ldots, 0) = 0$. We assume that the system is transformable, via a smooth non-singular state transformation, to the following

$$\dot{\mathbf{z}}_i = \mathbf{A}_i \mathbf{z}_i + \mathbf{B}_i u_i + \mathbf{f}_i(\mathbf{z}) \tag{2}$$

where $\mathbf{z} \in \mathbb{R}^{n_1 + \cdots + n_N}$ denotes the state of the overall system, and $(\mathbf{A}_i, \mathbf{B}_i)$ is in Brunovsky canonical form. We also assume that the interconnections satisfy the matching condition, i.e., $\mathbf{f}_i(\mathbf{z}) = \mathbf{B}_i w_i(\mathbf{z})$. Later in this paper, we will extend the controller to systems in strict feedback form. Suppose the interconnections are polynomially bounded as follows:

$$\|w_i(\mathbf{z})\| = \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ij}^k \|\mathbf{z}_j\|^k \tag{3}$$

The control objective is to design decentralized control laws that can achieve regulation of each subsystem state to zero. First, an example is considered and then the design is extended to the general system discussed above.

3 An Example

Consider the regulation problem for the following system composed of two subsystems:

$$\text{Subsystem 1: } \begin{aligned} \dot{z}_{11} &= z_{12} \\ \dot{z}_{12} &= a_{11} z_{11} z_{21} + a_{12} z_{21}^2 + u_1 \end{aligned} \tag{4}$$

$$\text{Subsystem 2: } \begin{aligned} \dot{z}_{21} &= z_{22} \\ \dot{z}_{22} &= a_{21} z_{12} z_{21} + a_{22} z_{11}^2 + u_2 \end{aligned} \tag{5}$$

where $a_{11}, a_{12}, a_{21}, a_{22}$ are constants. The subsystems can be written as

$$\dot{\mathbf{z}}_i = \mathbf{A}_i \mathbf{z}_i + \mathbf{B}_i u_i + \mathbf{B}_i w_i(\mathbf{z}), \quad i = 1, 2 \tag{6}$$

where $\mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $w_1(\mathbf{z}) = a_{11} z_{11} z_{21} + a_{12} z_{21}^2$, and $w_2(\mathbf{z}) = a_{21} z_{12} z_{21} + a_{22} z_{11}^2$. The interconnections $w_1(\mathbf{z})$ and $w_2(\mathbf{z})$ can be bounded similar to (3) with $N = 2$ and $p_{ij} = 2$ as follows:

$$\|w_1(\mathbf{z})\| \leq \beta_{11}^2 \|\mathbf{z}_1\|^2 + \beta_{12}^2 \|\mathbf{z}_2\|^2 \tag{7}$$

$$\|w_2(\mathbf{z})\| \leq \beta_{21}^2 \|\mathbf{z}_1\|^2 + \beta_{22}^2 \|\mathbf{z}_2\|^2 \tag{8}$$

For this example, we first design the decentralized control laws given in [2, 3] assuming that the bounds on coefficients of the polynomial bounds are known. The decentralized control laws for the system are:

$$u_i = -\alpha_i \mathbf{B}_i \mathbf{P}_i \mathbf{z}_i (1 + \|\mathbf{z}_i\|^{2(p-1)}) \tag{9}$$

where $p = \max_{i,j} p_{ij} = 2$ denotes the highest degree in the polynomial bound and $\mathbf{P}_i$ is the positive definite solution of the Algebraic Ricatti equation

$$\mathbf{A}_i \mathbf{P}_i + \mathbf{P}_i \mathbf{A}_i - 2\alpha_i \mathbf{P}_i \mathbf{B}_i \mathbf{B}_i^T \mathbf{P}_i + \mathbf{Q}_i = 0$$

By choosing α_i and $\mathbf{Q}_i$, we can solve the algebraic Ricatti equation to get $\mathbf{P}_i$. Stability is shown in [2, 3] by using the following Lyapunov function candidate

$$V(\mathbf{z}) = \sum_{i=1}^{N} \sum_{k=1}^{p} (\mathbf{z}_i^T P_i \mathbf{z}_i)^k \tag{10}$$

We refer the readers to [2, 3] for the proof. Notice that the control scheme uses cubic terms in the states to cancel out interconnecting nonlinearities, which are quadratically bounded. Furthermore, notice that the controller will use a $(2p-1)$th-order term to compensate for interconnecting nonlinearities that are pth-order bounded.

Now, we propose a lower-order decentralized controller for this example. Define the following signals: $s_1 = z_{12} + \lambda_1 z_{11}$ and $s_2 = z_{22} + \lambda_2 z_{21}$. Consider the following control laws:

$$u_i = -\gamma_i s_i - \lambda_i z_{i2} - \alpha_i \mathbf{sgn}(s_i) \|\mathbf{z}_i\|^p \tag{11}$$

Notice that only quadratic terms appear in the control laws. Consider the following Lyapunov function candidate, $V = |s_1| + |s_2|$. Differentiating along the trajectories of the closed-loop system, and choosing $\alpha_i \geq \beta_{1i}^2 + \beta_{2i}$, we obtain

$$\dot{V} \leq -\gamma_1 |s_1| - \gamma_2 |s_2| \tag{12}$$

Let $\gamma = \min\{\gamma_1, \gamma_2\}$.Then we obtain, $\dot{V}(t) \leq -\gamma V(t)$, which gives $V(t) \leq V(0) e^{-\gamma t}$, which implies that s_1 and s_2 converge to zero exponentially, and as a result z_{11}, z_{12}, z_{21} and z_{22} converge to zero exponentially.

Simulation results for the new decentralized controller (11) and the existing decentralized controller (9) are shown in Figures 2 and 1, respectively. From the closed-loop system responses shown in Figures 2 and 1, it should be noticed that the control input using (9) is much larger than that of the proposed controller, (11). In simulations, we have used $a_{ij} = 1.0$.

4 Controller Design

Now, we extend the controller design to the large-scale interconnected system given by (2). Define $s_i = z_{i,n_i} + \sum_{j=1}^{n_i - 1} \lambda_{i,n_i - j} z_{i,n_i - j}$.

Theorem 4.1 *For the interconnected system given by* (1) *that is transformed to* (2), *the following control laws, u_i, render the closed-loop system globally exponentially stable:*

$$u_i(t) = -\gamma_i s_i - \sum_{j=1}^{n_i - 1} \lambda_{i,n_i - j} z_{i,n_i - j + 1} - \alpha_i \mathbf{sgn}(s_i) \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \theta_{ij}^k \|\mathbf{z}_i\|^k \tag{13}$$

Proof: Consider the following Lyapunov function candidate,

$$V_i(\mathbf{z}_i) = |s_i| \tag{14}$$

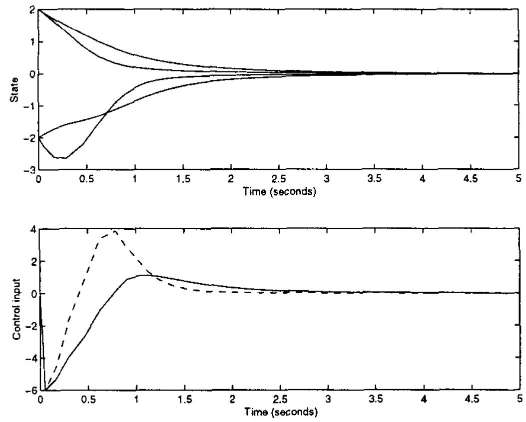

Figure 1: Response using the proposed controller (11)

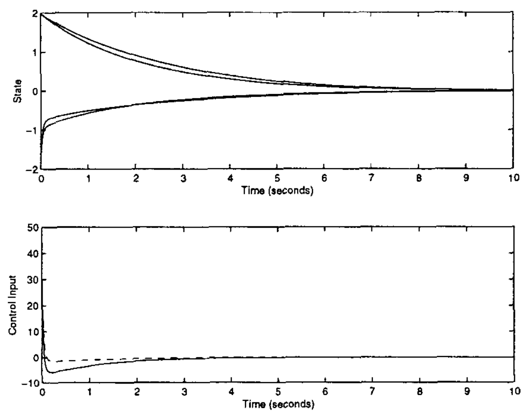

Figure 2: Response using controller (9)

Differentiating V_i along the trajectories of (2), we obtain

$$\dot{V}_i = -\gamma_i|s_i| + \mathbf{sgn}(s_i)w_i(\mathbf{z}) - \alpha_i \sum_{j=1}^{N}\sum_{k=1}^{p_{ji}} \theta_{ij}^k ||\mathbf{z}_i||^k \quad (15)$$

Using (3) for $w_i(\mathbf{z})$, we obtain

$$\dot{V}_i \leq -\gamma_i||\mathbf{z}_i||_1 - \sum_{j=1}^{N}\sum_{k=1}^{p_{ji}} \theta_{ij}^k ||\mathbf{z}_i||^k + \sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} \beta_{ij}^k ||\mathbf{z}_j||^k \quad (16)$$

Consider the composite Lyapunov function $V = \sum_{i=1}^{N} V_i$. Differentiating, we obtain

$$\dot{V} \leq -\sum_{i=1}^{N} \gamma_i|s_i| - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} (\theta_{ji}^k - \beta_{ij}^k)||\mathbf{z}_j||^k \quad (17)$$

Choosing $\theta_{ji}^k \geq \beta_{ij}^k$, and $\gamma = \min_i(\gamma_i)$, we obtain $\dot{V}(t) \leq -\gamma V(t)$. Hence, $V(t) \leq V(t_0)e^{-\gamma(t-t_0)}$. Thus, $\mathbf{z}_i$ converges to zero exponentially. ■

4.1 Adaptive Design

In this Section, we extend the robust controller designs to the adaptive case, wherein the controller gains are estimated. Again, we consider the example given in Section 2. Consider the following modification of the control law (11):

$$u_i = -\gamma_i s_i - \lambda_i x_{i2} - \widehat{\alpha}_i \mathbf{sgn}(s_i)||\mathbf{z}_i||^2 \quad (18)$$

where the $\widehat{\alpha}_i$ is the estimate of α_i. The update laws for $\widehat{\alpha}_i$ is given by

$$\dot{\widehat{\alpha}}_i = \eta_i||\mathbf{z}_i||^2 \quad (19)$$

where η_i is the adaptation gain. Now consider the following Lyapunov function candidate

$$V = |s_1| + |s_2| + \sum_{i=1}^{2} \frac{1}{2\eta_i}\widetilde{\alpha}_i^2 \quad (20)$$

where $\widetilde{\alpha}_i = \widehat{\alpha}_i - \alpha_i$. Differentiating along the trajectories of the closed-loop system, we obtain

$$\begin{aligned} \dot{V} \leq & -\gamma_1|s_1| - \gamma_2|s_2| + \sum_{i=1}^{2} \frac{1}{\eta_i}\widetilde{\alpha}_i\dot{\widehat{\alpha}}_i \\ & - \sum_{i=1}^{2} \widehat{\alpha}_i||\mathbf{z}_i||^2 + \sum_{i=1}^{2} \alpha_i||\mathbf{z}_i||^2 \end{aligned} \quad (21)$$

Let $\gamma = \min\{\gamma_1, \gamma_2\}$. Grouping terms together, we obtain,

$$\dot{V}(t) = -\gamma(|s_1| + |s_2|) + \sum_{i=1}^{2} \widetilde{\alpha}_i \left(\frac{1}{\eta_i}\dot{\widehat{\alpha}}_i - ||\mathbf{z}_i||^2\right) \quad (22)$$

Using the adaptation laws (19), we obtain

$$\dot{V}(t) = -\gamma(|s_1| + |s_2|) \quad (23)$$

Thus, $s_i, \widetilde{\alpha}_i \in L_\infty$. Notice that $\dot{V}(t)$ is absolutely continuous. Hence, invoking the nonsmooth Lyapunov theorem [7], we have $s_i \to 0$ as $t \to \infty$, which implies $\mathbf{z}_i \to 0$ as $t \to \infty$. Also, since the adaptation laws are given by (19), $\dot{\widehat{\alpha}}_i \to 0$ as $t \to \infty$. Simulation results for the adaptive case are shown in Figure 3. The third plot in Figure 3 gives the gain estimates, $\widehat{\alpha}_1$(solid) and $\widehat{\alpha}_1$(dashed). Simulations results verify the effectiveness of the proposed scheme for systems satisfying matching conditions. The technique shown in this example can be easily extended to construct adaptive decentralized controllers for the general large-scale system given by (2).

5 Systems in Strict Feedback Form

In this section, we show, via an example, that decentralized technique developed in the previous sections can be extended to large-scale systems in the strict feedback form. Consider the following example[3],

Subsystem 1:

$$\begin{aligned} \dot{y}_{11} &= y_{12} \\ \dot{y}_{12} &= x_{11} + \theta_1(\xi_{11}y_{11}y_{21} + \xi_{12}y_{21}^2) \\ \dot{x}_{11} &= u_1 + \theta_1 x_{11}y_{21}^2 \end{aligned} \quad (24)$$

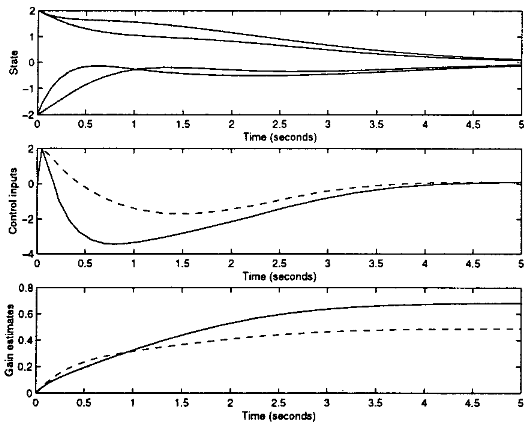

Figure 3: Decentralized adaptive control

Subsystem 2:

$$\begin{aligned} \dot{y}_{21} &= x_{21} + \theta_2(\xi_{21}y_{11}^2 + \xi_{22}y_{12}y_{21}) \\ \dot{x}_{21} &= u_2 + \theta_2 x_{21} y_{11}^2 \end{aligned} \quad (25)$$

Define $w_{10} := \theta_1(\xi_{11}y_{11}y_{21} + \xi_{12}y_{21}^2)$, $w_{11} := \theta_1 x_{11} y_{21}^2$, $w_{20} := \theta_2(\xi_{21}y_{11}^2 + \xi_{22}y_{12}y_{21})$, and $w_{21} := \theta_2 x_{21} y_{11}^2$. The nonlinear interconnections can be bounded as,

$$\|w_{10}\| \leq \alpha_{11}\|\mathbf{y}_1\|^2 + \alpha_{12}\|y_{21}\|^2 \quad (26)$$

$$\|w_{11}\| \leq \beta_{11}\|x_{11}\|^2 + \beta_{12}\|y_{21}\|^4 \quad (27)$$

$$\|w_{20}\| \leq \alpha_{21}\|\mathbf{y}_1\|^2 + \alpha_{22}\|y_{21}\|^2 \quad (28)$$

$$\|w_{21}\| \leq \beta_{21}\|x_{21}\|^2 + \beta_{22}\|\mathbf{y}_1\|^4 \quad (29)$$

where $\mathbf{y}_1 = [y_{11}, y_{12}]^T$. Define $s_{10} = y_{12} + \lambda_{10}y_{11}$, $s_{11} = x_{11} + \lambda_{11}s_{10}$, $s_{20} = y_{21}$, and $s_{21} = x_{21} + \lambda_{21}y_{21}$. Consider the following control laws:

$$\begin{aligned} u_1 = &-\sigma_{11}s_{11} - \sigma_{10}|s_{10}|\mathbf{sgn}(s_{11}) - (x_{11} + \lambda_{10}y_{12}) \cdot \\ &(\mathbf{sgn}(s_{10}) + \lambda_{11}\mathbf{sgn}(s_{11})\mathbf{sgn}(s_{11}) \quad (30) \\ &- \mathbf{sgn}(s_{11})\left(\widehat{\rho}_{11}^1\|\mathbf{y}_1\|^2 + \widehat{\rho}_{12}^2\|\mathbf{y}_1\|^4 + \widehat{\rho}_{13}\|x_{11}\|\right) \\ u_2 = &-\sigma_{11}s_{21} - \sigma_{20}|s_{20}|\mathbf{sgn}(s_{21}) \quad (31) \\ &- x_{21}(\mathbf{sgn}(s_{20}) + \lambda_{21}\mathbf{sgn}(s_{21})\mathbf{sgn}(s_{21}) \\ &- \mathbf{sgn}(s_{21})\left(\widehat{\rho}_{21}^1\|y_{21}\|^2 + \widehat{\rho}_{22}^2\|y_{21}\|^4 + \widehat{\rho}_{23}\|x_{21}\|\right) \end{aligned}$$

where σ's and $\widehat{\rho}$'s are controller gains. Consider the following Lyapunov function candidates for each subsystem: $V_1 = |s_{10}| + |s_{11}|$, $V_2 = |s_{20}| + |s_{21}|$. Stability of the closed-loop system for the proposed controller can be shown by using the following composite Lyapunov function candidate, $V = V_1 + V_2$. Differentiating the Lyapunov function along the trajectories of the system (24)-(25) and simplifying, we obtain

$$\begin{aligned} \dot{V} = &(x_{11} + \lambda_{10}y_{12} + w_{10})(\mathbf{sgn}(s_{10}) + \lambda_{11}s_{11}) \\ &+ (u_1 + w_{11})\mathbf{sgn}(s_{11}) \\ &+ (x_{21} + w_{20})(\mathbf{sgn}(s_{20}) + \lambda_{21}s_{21}) \\ &+ (u_2 + w_{21})\mathbf{sgn}(s_{21}) \end{aligned} \quad (32)$$

Using the control laws (30) and (31) and the bounds on $w_{10}, w_{11}, w_{20}, w_{21}$, and simplifying we obtain

$$\begin{aligned} \dot{V} = &-\sigma_{10}|s_{10}| - \sigma_{11}|s_{11}| - \sigma_{20}|s_{20}| - \sigma_{21}|s_{21}| \\ &- (\widehat{\rho}_{11} - \rho_{11})\|\mathbf{y}_1\|^2 - (\widehat{\rho}_{21} - \rho_{21})\|y_{21}\|^2 \\ &- (\widehat{\rho}_{12} - \rho_{12})\|\mathbf{y}_1\|^4 - (\widehat{\rho}_{22} - \rho_{21})\|y_{21}\|^4 \quad (33) \\ &- (\widehat{\rho}_{13} - \rho_{13})\|x_{11}\|^2 - (\widehat{\rho}_{23} - \rho_{23})\|x_{21}\|^2 \end{aligned}$$

where $\rho_{11} = (1 + \lambda_{11})\alpha_{11} + (1 + \lambda_{21})\alpha_{21})$, $\rho_{21} = (1 + \lambda_{11})\alpha_{12} + (1 + \lambda_{21})\alpha_{22})$, $\rho_{12} = \beta_{22}$, $\rho_{13} = \beta_{11}$, $\rho_{22} = \alpha_{22}$, and $\rho_{23} = \beta_{12}$. Choosing $\widehat{\rho}_{ij} \geq \rho_{ij}$, we obtain

$$\dot{V} \leq -\sigma_{10}|s_{10}| - \sigma_{11}|s_{11}| - \sigma_{20}|s_{20}| - \sigma_{21}|s_{21}|$$

Let $\sigma = \min\{\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21}\}$, then $\dot{V} \leq -\sigma V$. Thus, $V(t)$ converges to zero exponentially. Hence, $y_{11}, y_{12}, y_{21}, x_{11}, x_{21}$ converge to zero exponentially.

Next, we consider the adaptation of the controller gains, $\widehat{\rho}_{ij}$. Define $\widetilde{\rho}_{ij}(t) = \widehat{\rho}_{ij}(t) - \rho_{ij}$. Consider the following adaptation laws:

$$\dot{\widehat{\rho}}_{11} = \eta_{11}\|\mathbf{y}_1\|^2, \quad \dot{\widehat{\rho}}_{12} = \eta_{12}\|\mathbf{y}_1\|^4 \quad (34)$$

$$\dot{\widehat{\rho}}_{21} = \eta_{21}\|y_{21}\|^2, \quad \dot{\widehat{\rho}}_{22} = \eta_{22}\|y_{21}\|^4 \quad (35)$$

$$\dot{\widehat{\rho}}_{13} = \eta_{13}\|x_{11}\|^2, \quad \dot{\widehat{\rho}}_{23} = \eta_{23}\|x_{21}\|^2 \quad (36)$$

Consider the following modification of the Lyapunov function candidate V,

$$V_0 = V + \sum_{i=1}^{2}\sum_{j=1}^{3} \frac{1}{2\eta_{ij}}\widetilde{\rho}_{ij}^2 \quad (37)$$

Differentiating V_0 and utilizing the adaptation laws, we obtain

$$\dot{V}_0 = -\sigma_{10}|s_{10}| - \sigma_{11}|s_{11}| - \sigma_{20}|s_{20}| - \sigma_{21}|s_{21}|$$

Hence, $s_{10}, s_{11}, s_{20}, s_{21}, \widetilde{\rho}_{ij} \in L_\infty$. Using similar arguments as those in Section 4, we obtain asymptotic convergence of $s_{10}, s_{11}, s_{20}, s_{21}$ to zero, which implies convergence of $y_{11}, y_{12}, x_{11}, y_{21}, x_{21}$ to zero. Notice that the first term in the control inputs, (30) and (31), adds a linear damping term to the system. This is facilitated by the definition of the functions $s_{10}, s_{11}, s_{20}, s_{21}$. Simulation results for the adaptive case are shown in Figure 4. Initial conditions for all signals were chosen same as in Example 3.1 of [3].

It should be observed that the decentralized control laws and adaptation laws proposed here are of considerably lower-order than those given in [2, 3]. Extension of the controller design to general systems in strict feedback form can be easily made. It should be observed that in our designs we have not utilized the structure of the interconnecting nonlinearities as is done in [3], i.e., w_{10} and w_{20} are functions of states $\mathbf{y}$ only. We can use the techniques given here for systems in which the nonlinear interconnections w_{ij} need not have a specific structure such as in the strict feedback form.

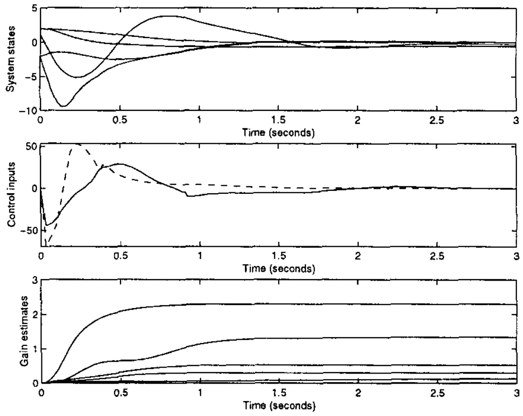

Figure 4: Closed-loop response: Strict feedback system example

6 Other Interconnections

Now we consider nonlinear interconnections that do not satisfy polynomial bounds of finite length. Suppose that a coupling nonlinearity is given by $w_i(\mathbf{x})$. If we can find functions $w_{i,1}(\mathbf{x}_1), w_{i,2}(\mathbf{x}_2), \dots, w_{i,N}(\mathbf{x}_N)$ such that

$$\|w_i(\mathbf{x})\| \leq \sum_{j=1}^{N} w_{i,j}(\mathbf{x}_j)$$

then it is possible to synthesize control algorithms using techniques that are described earlier in this paper. The motivation for doing this separation is that we can use the known individual function bounds $w_{i,j}(\mathbf{x}_j)$ in the decentralized control input u_i to compensate for functionally bound interconnections. The following examples will illustrate the idea and negate the need for giving a formal proof for the above cases. Consider the following nonlinear interconnections:

$$\begin{aligned} |x_1^{x_2}| &= |e^{x_2 \ln x_1}| leq e^{|x_2 \ln x_1|} \\ &\leq e^{(x_2^2 + (\ln x_1)^2)/2} = e^{x_2^2/2} e^{(\ln x_1)^2/2} \\ &\leq e^{x_2^2} + e^{(\ln x_1)^2} \end{aligned} \tag{38}$$

$$\begin{aligned} e^{x_1 x_2} &= 1 + x_1 x_2 + \frac{(x_1x_2)^2}{2!} + \frac{(x_1x_2)^3}{3!} + \cdots \\ &\leq 1 + \frac{1}{2}\left\{x_1^2 + \frac{x_1^4}{2!} + \frac{x_1^6}{3!} + \cdots\right\} \\ &\quad + \frac{1}{2}\left\{x_2^2 + \frac{x_2^4}{2!} + \frac{x_2^6}{3!} + \cdots\right\} \\ &\leq e^{x_1^2} + e^{x_2^2} \end{aligned} \tag{39}$$

We considered two nonlinear interconnections that cannot be bounded by finite length polynomials. The bounds as shown for these two interconnections can be separated into sum of other known functions. Each of the known function depends on individual subsystem states.

7 Conclusions

In this work, we developed a new decentralized control scheme for large-scale interconnected nonlinear systems with interconnections bounded by higher order polynomials in states. The contribution of this work to the existing literature is two fold: (a) the proposed decentralized control scheme is considerably lower order in states when compared to the existing decentralized control algorithms; (b) The control scheme is simple to design and is suitable for large-scale systems that do not satisfy matching conditions. In essence, the proposed decentralized scheme can be applied to a larger class of systems.

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