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Hybrid Force/Motion Control of Two Arms Carrying an Object

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Abstract

In this work we develop a hybrid force/motion controller for coordination of two robot arms carrying an object. A closed chain dynamic model is developed which helps in designing the motion controller independent of the force controller. A stable adaptive motion controller and a parameter adaptation law is designed to estimate the unknown parameters. A feedforward plus PI type force controller is developed. Stability analysis is done for both motion and force controller. It is shown that the motion controller is robust to bounded disturbances and the force controller is robust to small delay in force feedback. The theoretical results have been validated by simulations.

1. Introduction

Considerable research has been done in recent years in the area of coordinated control of multiple manipulators. Some of the several applications of multiple manipulators in industry are operations such as materials handling and assembly of parts. Also a multiple manipulator system can handle heavier loads than compared to a single manipulator. In this paper we address the problem of two manipulators moving an object along a desired path, while exerting desired internal forces on the object.

Recently, several control strategies for coordinated control of multiple manipulators have been presented. Tarn and Bejczy[7] used a non-linear coordinate transformation to obtain an exact linear decoupled system. Zheng and Luh[5] considered the two manipulators as a closed chain and derived a set of holonomic constraints, which are satisfied by the position and orientation of the two manipulators when the closed chain is in motion. Hu and Goldenberg[6] developed three subsystem error equations using the dynamic equations, i.e., position error subsystem, a contact force error subsystem, and an internal force error subsystem, and they used an adaptive scheme to control these subsytems. Yao and Tomizuka[14] developed an adaptive controller for multiple manipulators handling a constrained object. This controller is shown to be robust to bounded velocity and force measurement noise. The problem of single robot arm in constrained motion is studied by many researchers, [e.g., 4,10, and 15].

Of all the schemes considered none of them explicitly show that the motion controller is independent of the force controller. In this paper we develop a dynamic model which helps in decoupling the motion control law from the force control law in the sense that force control does not affect motion control, but the motion errors do affect the internal force control. Thus, the motion error subsystem being independent of the force terms gives the advantage of designing force and motion controllers separately. Uncertainty in the parameters of the manipulators and that of the object is considered. An adaptive controller has been developed to estimate the mass, inertia of the object and the unknown parameters of the manipulators. Robustness of the proposed controller to bounded disturbances is shown.

A brief outline of the paper is as follows. In section 2 we derive the dynamic equations of the closed chain from the dynamic equations of the manipulators and the object. In section 3 the control law and the adaptation law are derived. Using this motion control law and the adaptation law, stability is proved. In section 4 we give a force control strategy, which contains the feedforward control term and the PI control terms.The force controller is shown to be robust to small delay in force feedback. In section 5 we present some typical simulation results. Conclusions are given in section 6.

2. Dynamic Equations

Consider two rigid manipulators, the dynamic equations of which in joint coordinates using the Lagrangian formulation is of the form

$$M_i(q_i)\ddot{q_i} + C_i(q_i,\dot{q}_i)\dot{q_i} + G_i(q_i) + J_i^{\top}(q_i)F_i + d_i(t) = \tau_i \quad (1)$$

where the subscript i (i=a,b) denotes the i-th manipulator, $q_i \in \Re^{n\times 1}$ are the joint angles, $\tau_i \in \Re^{n\times 1}$ is the vector of generalized joint torques. $M_i(q_i) \in \Re^{n\times n}$ is the inertia matrix of the i-th manipulator, $C_i(q_i,\dot{q}_i) \in \Re^{n\times n}$ is the matrix of centripetal and Coriolis forces of i-th manipulator, $G_i \in \Re^{n\times 1}$ is the gravity force vector, $J_i^{\top}(q_i) \in \Re^{n\times n}$ is the Jacobian matrix that transforms the generalized contact force in the world coordinate system to the joint space, $F_i \in \Re^{n\times 1}$ is the generalized force applied by the i-th manipulator on the object, and $d_i(t)$ is the disturbance vector. Notice that the number of joints in each manipulator is equal to the dimension of the generalized force vector. This implies that n is six for manipulators in three dimensional work space with independent rotation and orientation, and it is three for planar motion in the x-y plane with an independent rotational freedom in the z-direction.

We assume the following in deriving the closed chain equations: 1) The manipulators firmly grasp the object so that the contact point of the manipulators end-effector and the object remains same throughout the motion i.e., there is no relative motion between the object and the end-effectors. 2) Object is rigid i.e., object does not get deformed with the application of internal forces. These assumptions imply that the two manipulators and the rigid object is making a closed kinematic chain, and the last link of the two manipulators and the object become one link in the chain. Fig. 1 shows a schematic of two robot arms holding an object.

Now we write the dynamic equations of the two manipulators compactly as

$$M_t(q_t)\ddot{q}_t + C_t(q_t, \dot{q}_t)\dot{q}_t + G_t(q_t) + J_t^\top F + d_t(t) = \tau \quad (2)$$

where $M_t = diag(M_a, M_b)$, $C_t = diag(C_a, C_b)$, $G_t = (G_a^\top, G_b^\top)^\top$, $J_t = diag(J_a, J_b)$, $d_t = (d_a^\top, d_b^\top)$, $\tau = (\tau_a^\top, \tau_b^\top)^\top$, $F = (F_a^\top, F_b^\top)^\top$, and $q_t = (q_a^\top, q_b^\top)^\top$

The following are the Newton-Euler equations of the freebody of the object

$$M_o(x_o)\ddot{x}_o + C_o(x_o, \dot{x}_o)\dot{x}_o + G_o(x_o) + d_o(t) = F_o \quad (3)$$

where $M_o(x_o) \in \Re^{n\times n}$ is the inertia matrix of the object, $x_o \in \Re^{n\times 1}$ is the position vector of the object, $C_o(x_o, \dot{x_o}) \in \Re^{n\times n}$ contains the Coriolis cross product terms of the object, and $F_o \in \Re^{n\times 1}$ is the force acting at the center of mass of the object. The force acting at the center of mass of the object is $F_o = \sum_i D_i^\top F_i = D^\top F$, where $D^\top = [D_a^\top, D_b^\top]$, $F_i \in \Re^{n\times 1}$, is the generalized force exerted by manipulator i and $D_i^\top \in \Re^{n\times n}$, is the matrix that transforms the generalized force of manipulator i to its equivalent force at the center of mass of the object.

The manipulators are kinematically coupled with the object in between them so that we can quantitatively relate the generalized coordinates and velocities of the manipulators through the kinematic constraints. The manipulator Jacobian relates the velocity of end-effector i in cartesian coordinates to the joint space and the matrix D_i relates the velocity of end-effector i to the velocity of the center of mass of the object. These equations are as follows

$$\dot{x}_i = J_i(q_i)\,\dot{q}_i \;\; and \;\; \dot{x}_i = D_i\dot{x}_o \quad (4)$$

Defining $\dot{x}^\top = [\dot{x}_a^\top, \dot{x}_b^\top]^\top$, we obtain

$$\dot{x} = J_t^\top \dot{q}_t \;\; and \;\; \dot{x} = D\dot{x}_o \quad (5)$$

From equation (6) we can write the velocity of the object as

$$\dot{x}_o = D^+\dot{x} = D^+J_t\dot{q}_t = A\dot{q}_t \quad (6)$$

where $A = D^+J_t$ and D^+ is the generalized inverse. Differentiating equation (6) with respect to time, we obtain the acceleration of the object

$$\ddot{x}_o = A\ddot{q}_t + \dot{A}\dot{q}_t \quad (7)$$

The force at the object center of mass is given by $F_o = D^\top F$. Matrix $D^\top$ has full rank and posseses a non-trivial null space. Since $F_o = D^\top F$, a force in the null space of $D^\top$ does not contribute to the motion of the object but only to the buildup of internal forces. Let P be the matrix whose columns span the null space of $D^\top$, then we have $P \in \Re^{2n\times n}$ and $D^\top P \equiv 0$. Given a force F we can write it as a component which lies in the range space of the matrix $D^\top$ and a component which lies in the null space of $D^\top$ as

$$F = D^{+T}F_o + Pf_{int} \quad (8)$$

Using dynamic equations of the object (3) we can write (9) as

$$F = D^{+\top}(M_o(x_o)\ddot{x}_o + C_o(x_o, \dot{x}_o) + G_o(x_o) + d_o(t)) + Pf_{int} \quad (9)$$

When the manipulators rigidly grasp the object, in position subspace there are n natural constraints. So the effective degrees of freedom of the manipulator system reduces to n. Therefore n generalized coordinates are sufficient to describe the closed chain motion of the manipulator system. We can choose n joint variables as independent generalized coordinates and the other n joint variables depend on the independent generalized coordinates. Let the n independent generalized coordinates be given by $q = [q_1, q_2, \cdots, q_n]$, without loss of generality we can choose these n components as the first n components of q_t. Let the other n dependent coordinates be given by $q_c = [q_{c1}, q_{c2}, \cdots, q_{cn}]$. We can express the dependent coordinates as a function of independent coordinates as $q_c = \Omega(q)$ [4]. Differentiating this with respect to time yields

$$\dot{q}_c = T_1(q)\dot{q}, \quad \ddot{q}_c = T_1(q)\ddot{q} + \dot{T}_1(q)\dot{q} \quad (10)$$

where $T_1(q) = \partial\Omega(q)/\partial q$. Define a tranformation $T(q)^\top = [I_{n\times n}; T_1^\top]^\top$. Now we can write the velocity and acceleration of the joint coordinates, q_t, in terms of the independent coordinates q as

$$\dot{q}_t = T(q)\dot{q}; \quad \ddot{q}_t = T(q)\ddot{q} + \dot{T}(q)\dot{q} \quad (11)$$

Substituting the above in the dynamic equations of the manipulators (2), we obtain

$$M(q)T(q)\ddot{q} + (M(q)\dot{T}(q) + C(q,\dot{q})T(q))\dot{q} + G(q) + J_t^\top(q)F + d_t(t) = \tau \quad (12)$$

Note that in the following equations we drop the arguments of the matrices wherever it is clear from the context. Using the transformation we can write the kinematic constraint's in terms of the independent generalized coordinates as,

$$\dot{x}_o = D^+\dot{x} = D^+J_t(q)T(q)\dot{q} = A(q)T(q)\dot{q} \quad (13)$$

$$\ddot{x}_o = \dot{A}(q)T(q)\dot{q} + A(q)\dot{T}(q)\dot{q} + A(q)T(q)\ddot{q} \quad (14)$$

Premultiplying equation (9) with $J_t^\top$, we obtain

$$J_t^\top F = A^\top [M_o \dot{A} T \dot{q} + M_o A \dot{T} \dot{q} + M_o A T \ddot{q} + C_o A T \dot{q} + G_o + d_o(t)] + J_t^\top P f_{int} \quad (15)$$

Substituting (15) in the dynamic equations (12), we obtain the closed form dynamic equations containing the object and the manipulators dynamics as

$$H(q)\ddot{q} + N(q,\dot{q})\dot{q} + G(q) + B^\top(q) f + d(t) = \tau \quad (16)$$

where $H(q) = [M + A^\top M_o A]T$, $N(q,\dot{q}) = [A^\top M_o \dot{A} + A^\top C_o A + C]T + [A^\top M_o A + M]\dot{T}$, $G = [G_o + G_t]$, $B^\top = J_t^\top P$, *and* d(t)=$d_t(t) + A^\top d_o(t)$

This is the equation of the closed chain obtained after all the constraints are embedded into the original dynamic equations of the manipulators. Note that the matrices H *and* N are not square and the dimension of which is $2n \times n$. Also note that $q \in \Re^{n\times 1}$ *and* $\tau \in \Re^{2n\times 1}$.

Using the kinematic constraints we can show that $T^\top B^\top = 0$. Premultiplying equation (5) with $P^\top$, we get

$$P^\top \dot{x} = P^\top D \dot{x}_o = [D^\top P]^\top \dot{x}_o \quad (17)$$

Since P is the null space of $D^\top$ and $\dot{x} = J_t^\top T \dot{q}$, the above equation becomes $P^\top \dot{x} = P^\top J T \dot{q} = 0$. Noting that q is an independent set of coordinates and $B = P^\top J$, we obtain

$$T^\top B^\top = 0 \quad (18)$$

This is the constraint on the closed kinematic chain and will be utilized later to show that the motion error equation is independent of the internal force terms.

3. Control Law

Before we proceed with designing the adaptive controller for the closed chain equations we state some properties which will be used during the stability analysis of the proposed controller. 1) $\dot{M}_i(q) - 2C_i(q,\dot{q})$ is skew symmetric for $i = a$ *or* b which implies, $(\dot{M}_t - 2C_t)$ is skew symmetric. 2) The inertia matrix of each manipulator is positive definite and bounded from below and above i.e., there exists $\nu_i > 0$ and $\gamma_i > 0$ such that $\nu_i I_n \le M_i(q_i) \le \gamma_i I_n$, for every $q_i \in \Re^{n\times 1}$, where $i = a$ *or* b. 3) The dynamic structure of the object and each manipulator is linear in terms of the suitably selected set of unknown parameters.

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = W_i(q,\dot{q},\ddot{q})\Theta_i$$
$$M_o(x_o)\ddot{x}_o + C_o(x_o,\dot{x}_o)\dot{x}_o + G_o(x_o) = W_o(x_o,\dot{x}_o,\ddot{x}_o)\Theta_o$$

where $i = a$ *or* b, $W_i \in \Re^{n\times r}$ is the known regressor matrix and $\Theta_i \in \Re^{r\times 1}$ is the vector of unknown parameters in the case of manipulators and $W_o \in \Re^{n\times r_o}$ is the known matrix and $\Theta_o \in \Re^{r_o\times 1}$ is the vector of unknown parameters of the object. Now define the matrix $E(q) = T^\top(q)H(q)$, then $\dot{E}(q) - 2T^\top N$ is skew symmetric. The proof of this is quite procedural and can be shown by taking the derivative of $E(q)$ and using the properties stated above.

We define the following errors and reference velocities, $e_p = q_d - q$, is the joint position error, $e_f = f_d - f$, is the force error, $\dot{q}_r = \dot{q}_d + \Lambda_p e_p$, is the joint reference velocity, $\ddot{q}_r = \ddot{q}_d + \Lambda_p \dot{e}_p$, is the joint reference acceleration, $e_v = \dot{q}_r - \dot{q} = \dot{e}_p + \Lambda_p e_p$, is the reference velocity error, where Λ_p is a positive definite diagonal matrix. Using properties 1 and 2 and equation (??), we can write

$$H(q)\ddot{q}_r + N(q,\dot{q})\dot{q}_r + G(q) = W(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta \quad (19)$$

where $W(\cdot)$ is the known regressor matrix and Θ is the unknown parameter vector containing the unknown parameters of the two manipulators and the object. Before stating the control law, we impose bounds on the disturbances in the manipulators $d_t(t)$, and the object $d_o(t)$ as

$$|d_t(t)| \le \delta d_t(t) \ \ and \ \ |d_o(t)| \le \delta d_o(t) \quad (20)$$

We write the control law as $\tau = \tau_m + \tau_c$, where τ_m represents the motion control law and τ_c the force control law. We choose the adaptive motion control law and the parameter adaptation law as

$$\tau_m = W(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} + \gamma T e_v + \delta d(t) \odot T\ sat(e_v) \quad (21)$$

$$\dot{\hat{\Theta}} = -\Gamma W^\top(q,\dot{q},\dot{q}_r,\ddot{q}_r) T^\top e_v \quad (22)$$

where $\hat{\Theta}$ is the vector of estimated parameters and $sat(\cdot)$ represents the saturation function [10], and $a \odot b$ represents component-wise multiplication of two vectors a and b, and $\delta d(t) = \delta d_t(t) + |A^\top| \delta d_o(t)$. The force control law will be discussed later and has the form $B^\top f_c$. The above control law can be substituted into the closed chain equations (??) to get,

$$H\ddot{q} + N\dot{q} + G + B^\top f + d(t) =$$
$$W(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} + \gamma T e_v + B^\top f_c + \delta d(t) \odot T sat(e_v)$$

We can modify this by using the reference velocity error as defined previously to obtain the error equation,

$$H\dot{e}_v + N e_v = W(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\Theta} - \gamma T e_v - B^\top(f_c - f) + d(t) - \delta d(t) \odot T sat(e_v)$$

Premultiplying this with $T^\top$ and knowing that $T^\top B^\top \equiv 0$, we obtain the motion error equation which does not contain any force terms.

$$E\dot{e}_v = -T^\top N e_v + T^\top W \tilde{\Theta} - T^\top T e_v + T^\top d(t) - T^\top \delta d \odot T sat(e_v) \quad (23)$$

Theorem 1: *With the control law (??) and the parameter adaptation law (20) and with the bounds on the disturbance, (??),* $e_v, e_p, \dot{e}_p \to 0$ *asymptotically.*

Proof: Choose a Lyapunov function candidate as

$$V(t, e_v, \tilde{\Theta}) = \frac{1}{2}\{e_v^\top E(q) e_v + \tilde{\Theta}^\top \Gamma^{-1} \tilde{\Theta}\} \quad (24)$$

Taking the time derivative of equation (22) and using the skew symmetry property and the adaptation law, equation (20), we obtain

$$\dot{V}(t,e_v) = -\gamma e_v^\top T^\top T e_v + e_v^\top T^\top\{d(t) - \delta d(t) \odot T sat(e_v)\} \quad (25)$$

Now with $\delta d(t)$ as defined previously,

$$e_v^\top T^\top d(t) - e_v^\top T^\top\{\delta d(t) \odot T sate_v\} \leq 0 \quad (26)$$

From equation (23) and equation (26) and denoting σ_v as the smallest eigenvalue of $T^\top T$, we get

$$\dot{V}(t,e_v) \leq -\gamma e_v^T T^\top T e_v$$
$$\dot{V}(t,e_v) \leq -\gamma\sigma_v \|e_v\|^2$$
$$\Rightarrow \gamma\sigma_v \int_o^t \|e_v\|^2 d\sigma \leq V(e_v,0) - V(e_v,t) < \infty$$

From the above development we can see that $e_v \in L_2 \cap L_\infty$ and $\tilde{\Theta} \in L_\infty$. Also using the error equation $\dot{e}_v \in L_\infty$, since all the other terms in the error equation are bounded. As a result of this $e_v \to 0$, by Barbalat's lemma. Now the vector equation, $e_v = \dot{e}_p + \Lambda_p e_p$, can be written in Laplace domain as $e_p = G(s)e_v$, where $G(s) = [sI + \Lambda_p]^{-1}$. Since $e_v \in L_2 \cap L_\infty$ and $G(s)$ is exponentially stable and strictly proper transfer function matrix, it follows from [16] that $e_p \in L_2 \cap L_\infty$ and $\dot{e}_p \in L_2 \cap L_\infty$. So from Barbalat's lemma, $e_p \to 0$.

4. Force control law

The force control law is of the form $\tau_c = B^\top f_c$. The closed loop error equation (21) can be written as

$$B^\top(f_c - f) = \beta(e_v, \dot{e}_p, e_p, \tilde{\Theta}, d(t))$$

or

$$f_c - f = \alpha(e_v, \dot{e}_p, e_p, \tilde{\Theta}, d(t)) \quad (27)$$

where $\alpha(e_v, \dot{e}_p, e_p, \tilde{\Theta}, d(t)) = B^+\beta(e_v, \dot{e}_p, e_p, \tilde{\Theta}, d(t))$.

We choose the following force controller

$$f_c = f_d + K_p(f_d - f) + K_i \int_o^t (f_d - f) \quad (28)$$

where the first term is the feedforward term, second is the proportional term and the third is the integral term. The proportional and integral gain matrices, K_p and K_i, are positive definite and diagonal. Also the eigenvalues of K_p have to be bounded by 1, if this is not the case then we will see in the next section that a small delay in force feedback leads to instability of the system. Note that in the following equations we drop the arguments of α. Combining (27) and (28),

$$e_f + K_p e_f + K_i \int_o^t e_f = \alpha \quad (29)$$

Writing this equation in Laplace domain

$$\{(I + K_p)s + K_i\}e_f = s\alpha$$

In the above equation α is a function of the system variables $e_v, e_p, \dot{e}_p, \tilde{\Theta}$, and since the derivative of these system variables is bounded, the derivative of α is bounded. Since $\dot{\alpha}$ is bounded it follows from [16] that e_f is bounded and moreover if the motion errors converge to zero then the force error e_f converges to zero.

Delay in force feedback

Since a small delay in the force feedback is expected, the force controller should be robust to these delays. Denote the delay time by $\triangle t$. We can write the force control law as

$$f_c(t) = f_d(t) + K_p(f_d(t-\triangle t) - f(t-\triangle t)) + K_i \int_o^t (f_d(t-\triangle t) - f(\triangle t))$$

Notice that f_d being a desired force trajectory is available to us. Now with this force control law, equation (29) can be written as

$$e_f(t) - K_p e_f(t-\triangle t) + K_i \int_0^t e_f(p-\triangle t)dp = \alpha \quad (30)$$

In Laplace domain (30) is

$$[I + e^{-\triangle ts}\frac{(K_p s + K_i)}{s}]e_f = \alpha \quad (31)$$

Writing (31) componentwise as

$$1 + \frac{(kp_l s + ki_l)}{s}e^{-\triangle ts} = 0 \quad (32)$$

Define

$$G(s) = \frac{(kp_l s + ki_l)}{s}e^{-\triangle ts} \; and \; G_1(s) = \frac{(kp_l s + ki_l)}{s}$$

In the frequency response of the transfer functions $G(j\omega)$ and $G_1(j\omega)$, the magnitude of both transfer functions is the same, but the phase is different. The phase of the transfer function $G_1(j\omega)$ starts from -90 degrees and goes to zero as ω is increased. The frequency response plot of $G_1(j\omega)$ is shown in Fig. 2 for some typical values of kp_l and ki_l. The phase of pure delay $e^{-\triangle ts}$ decreases linearly as a function of ω. For $G(j\omega)$, stability is guaranteed if at $\omega = \omega_g$, the following condition is satisfied,

$$\angle G_1(j\omega_g) - \triangle t\omega_g > -180 \quad (33)$$

where ω is the frequency and ω_g is the gain crossover frequency. It can be seen that for $kp_l > 1$, a delay in force feedback always leads to instability of the system. For $kp_l < 1$, if the condition (33) is satisfied for the delay then the system is stable. So when $kp_l < 1$ and delay in force feedback is small so that condition (33) is satisfied and the motion errors converge to zero then $e_f \to 0$.

5. Simulation Results

Simulations were conducted using the parameters of the 3-link SCARA type planar arms. The trajectory of the object center of mass is $\frac{1}{2}x^2 + y^2 = r^2$ with

$x = -r\cos(\pi t/3)$ *and* $r = 0.1$ *m*. The desired internal forces on the object were chosen to be $[f_x, f_y, n_z] = [10+2\sin(\pi t/3), 10+2\sin(\pi t/3), 0]$. Initially the robot arms are assumed to be grasping the object at the initial point of the desired trajectory. The unknown disturbances in the arms and the object are assumed to be $d_i(t) = [2+\frac{1}{2}\sin(\pi t/2), 2+\frac{1}{2}\sin(\pi t/2), 0]^T$, $i = o, a, b$. Fig.3 and Fig.4 show the position and internal force errors respectively. Fig. 5 shows the estimated mass and inertia of the object. The true values of the mass and inertia of the object were assumed to be $20kg$ and $5kg - m^2$ respectively.

6. Conclusions

A dynamic model has been presented for two robot arms carrying an object. It has been shown that this dynamic model helps in designing the motion controller independent of the force controller. First a stable adaptive motion controller is designed and this is shown to be robust to bounded disturbances in the arms and the object. Then a feedforward plus PI type force controller is developed and is shown to be robust to small delays in force feedback when the eigenvalues of the position gain matrix are upper bounded by one and with a proper choice of integral gain matrix. Simulation results validate the proposed controller.

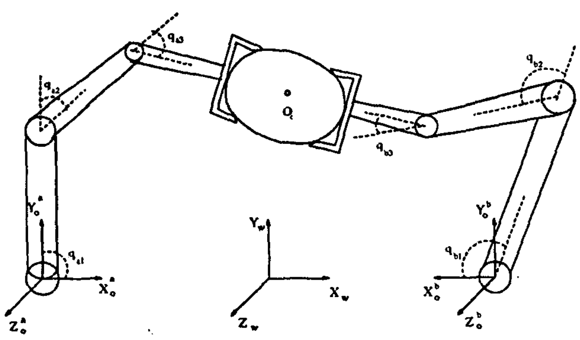

Figure 1: Two robot arms holding an object

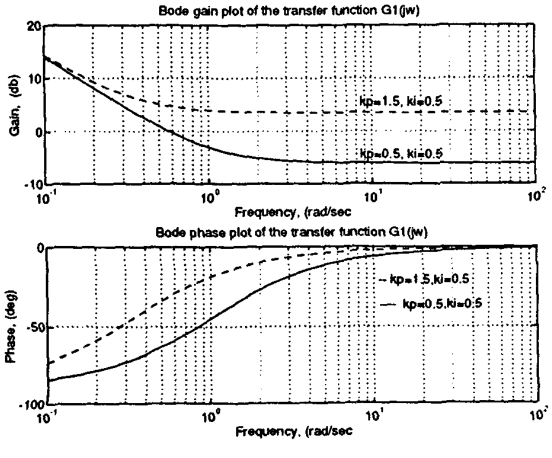

Figure 2: Frequency response plot of $G_1(jw)$.

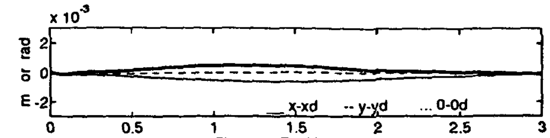

Figure 3: Position erorrs

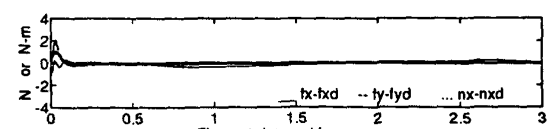

Figure 4: Internal force erorrs

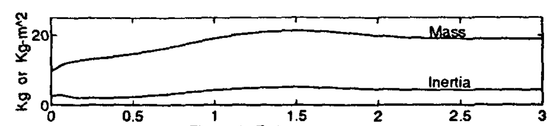

Figure 5: Estimated parameters

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