Adaptive Control of Two Robot Arms Carrying an Unknown Object [1]

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Abstract

In this work we consider the control of two robot manipulators carrying an unknown object. We consider the uncertainties in mass, inertia and center of mass of the object. A dynamic model is obtained for the combined system. An adaptive scheme is proposed to track the desired motion trajectory of the object in the presence of uncertainties. A feedforward plus PI type force controller is designed for internal force control. A Lyapunov based stability analysis is conducted for the proposed scheme. In the adaptive motion control scheme for the combined system we cannot specify the individual torques of the manipulators uniquely. To obtain a set of individual torques an optimal control problem is formulated by minimizing a cost function.

1. Introduction

Extensive research has been done in the control of multiple manipulators carrying an object. In the work done so far the control algorithms either assume that the object parameters as known or that only the inertial parameters as unknown [12],[6],[7],[10]. The algorithms proposed so far assume that the center of mass is exactly known. In this work we propose a control algorithm where we assume both the inertial parameters, mass and moment of inertia, and the center of mass of the object as unknown. The center of mass of the object being uncertain implies that the elements of the grasp matrix are uncertain. The grasp matrix maps forces at the contact points to the forces at the center of mass of the object. Although the grasp matrix depends on the location of the center of mass, the null space of the grasp matrix only depends on the location of the contact points on the object. We design an adaptive motion controller and parameter adaptation laws for estimating the uncertain parameters of the object. A feedforward plus PI type force controller is designed for internal force control. From the proposed adaptive scheme of the combined system there is no unique way of specifying the individual torques of the manipulators. To obtain individual torques an optimal control problem is formulated by minimizing a cost function.

The rest of the paper is organized as follows. Section 2 gives the notation used and the kinematics of the system of multiple manipulators carrying an object. In section 2.3 we show the independence of the null space of the grasp matrix from the position of the center of mass. In section 3 we derive the dynamics of the combined system using the kinematic constraints. An adaptive controller and parameter adaptation laws are designed in section 4 and an optimal control problem is formulated. In section 5 we give conclusions.

2. Notation and Kinematics

2.1. Notation

Here we give the notation which will be used throughout the paper. Figure 2.1 shows the notation of the coordinate systems.

$\{O_b X_b Y_b Z_b\}$ Base frame

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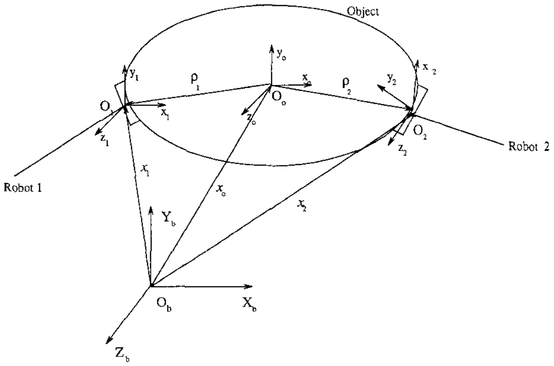

Figure 1: Notation of Coordinate Systems

$\{O_oX_oY_oZ_o\}$			Object frame
$\{O_iX_iY_iZ_i\}$			End effector frame of the i-th manipulator
x_i	$\in$	$\Re^3$	Position of the i-th end-effector
ω_i	$\in$	$\Re^3$	Angular velocity of the i-th end-effector
x_o	$\in$	$\Re^3$	Position of the object c.m.
ρ_i	$\in$	$\Re^3$	Position of the i-th contact point w.r.t $\{O_oX_oY_oZ_o\}$
ω_o	$\in$	$\Re^3$	Angular velocity of the object
f_{ci}	$\in$	$\Re^3$	Force applied by the i-th manipulator at the contact
n_{ci}	$\in$	$\Re^3$	Moment applied by the i-th manipulator at the contact
f_o	$\in$	$\Re^3$	Effective force at the object center of mass due to f_{c1} and f_{c1}
n_o	$\in$	$\Re^3$	Effective moment at the object center of mass due to n_{c1} and n_{c1}
I_3	$\in$	$\Re^{3\times 3}$	Denotes 3×3 identity matrix
O_3	$\in$	$\Re^{3\times 3}$	Denotes 3×3 zero matrix

2.2. Kinematics

In this section we develop the kinematics of the system and derive the kinematic constraints associated with the manipulator system grasping the object. In deriving the constraints we assume (i) the contact type between each end effector and the object to be rigid i.e., there is no relative motion between the end effector and the object (ii) the manipulators are non-redundant i.e., the number of joints in each manipulator is same as the degrees of freedom of the object. We derive the controller assuming that the object has six degrees of freedom ie., free to move in three dimensional space. This means that each of the manipulators has six degrees of freedom. The effective force and moment at the center of mass of the object due to forces applied by the individual manipulators is given by,

$$f_o = \sum_{i=1}^{2} f_{ci} \tag{2.1}$$

$$n_o = \sum_{i=1}^{2} (n_{ci} + \rho_i \times f_{ci}) \tag{2.2}$$

We can express this in matrix form by defining $F_o^\top = [f_o^\top, n_o^\top]$, $F_c^\top = [f_{c1}^\top, n_{c1}^\top, f_{c2}^\top, n_{c2}^\top]$, $Q_i = [\rho_i \times]$, $W_i = \begin{bmatrix} I_3 & O_3 \\ Q_i & I_3 \end{bmatrix}$, and $W = [W_1, W_2]$, as follows

$$F_o = WF_c \tag{2.3}$$

A contact force F_c can be written as sum of two component forces, one lying in the range space of the grasp matrix W, and the other lying in the null space of the grasp matrix,

$$F_c = W^{+}F_o + Pf_{int} \tag{2.4}$$

where W^{+} denotes the right pseudoinverse of the grasp matrix and the matrix P has as its coloumns the null vectors of the grasp matrix. Now we define the generalized velocity of each of the manipulators and the object as $\dot{z}_i^\top = [\dot{x}_i^\top, \omega_i^\top] and \dot{z}_o^\top = [\dot{x}_o^\top, \omega_o^\top]$. Dual to the force relationship we have the relationship between the end effector velocities and the object velocities as

$$\dot{z}_1 = W_1^\top \dot{z}_o, \quad \dot{z}_2 = W_2^\top \dot{z}_o \tag{2.5}$$

Combining these two equations by defining $\dot{z}^\top = [\dot{z}_1^\top, \dot{z}_2^\top]$ we obtain,

$$\dot{z} = W^\top \dot{z}_o \tag{2.6}$$

2.3. Null Space of the Grasp Matrix

The null space of the grasp matrix is $P = [I - W^{+}W]$, where $W^{+} = W^\top[WW^\top]^{-1}$. We consider the following form of the right generalized inverse of the grasp matrix given in [2] to compute the null space and to show that it is independent of the location of the center of mass of the object. The right generalized inverse of the grasp matrix weighted by a matrix A is given by $W_A^{+} = AW^\top[WAW^\top]^{-1}$. Notice that this definition of the right inverse means $WW_A^{-1} = I$. Using the following A matrix, we obtain the right generalized inverse and the corresponding null space of the grasp matrix, $P = [I - W_A^{+}W_A]$, as

$$A = \begin{bmatrix} O_3 & \alpha I_3 & O_3 & O_3 \\ \alpha I_3 & O_3 & O_3 & O_3 \\ O_3 & O_3 & O_3 & \alpha I_3 \\ O_3 & O_3 & \alpha I_3 & O_3 \end{bmatrix}$$

$$W_A^+ = \frac{1}{2}\begin{bmatrix} I_3 & O_3 \\ -Q_1 & I_3 \\ I_3 & O_3 \\ -Q_2 & I_3 \end{bmatrix}$$

$$P = \frac{1}{2}\begin{bmatrix} I_3 & O_3 & -I_3 & O_3 \\ O_3 & I_3 & Q_1 - Q_2 & -I_3 \\ -I_3 & O_3 & I_3 & O_3 \\ Q_2 - Q_1 & -I_3 & O_3 & I_3 \end{bmatrix}$$

Notice that the null space of the grasp matrix has the difference of the grasp matrices due to each manipulator. This difference ($Q_o := Q_2 - Q_1$) does not depend on the center of mass of the object but only the contact points of each manipulator on the object, which is known. So the null space can be computed without knowing the center of mass of the object. From now on we will denote W_A^+ by W^+.

3. Dynamics

The object dynamics from the Newton and Euler equations is of the form,

$$M_o(z_o)\ddot{z}_o + C_o(z_o, \dot{z}_o)\dot{z}_o = F_o \qquad (3.1)$$

where $M_o \in \Re^{6\times 6}$ is the symmetric positive definite inertia matrix and $C_o \in \Re^{6\times 6}$ is the matrix consisting of Coriolis and centrifugal terms. The arms dynamics in the cartesian coordinates is

$$M_i(z_i)\ddot{z}_i + C_i(z_i, \dot{z}_i)\dot{z}_i + f_{ci} = u_i \qquad (3.2)$$

where $i = 1, 2$, M_i is the symmetric positive definite inertia matrix and C_i is the matrix consisting of Coriolis and the centrifugal terms. We also assume that each manipulators and the object dynamics are derived such that the matrices $\dot{M}_o - 2C_o$ and $\dot{M}_i - 2C_i$ are skew symmetric. The dynamics of both the arms in combined form is

$$M(z)\ddot{z} + C(z, \dot{z})\dot{z} + F_c = u \qquad (3.3)$$

where $M = diag(M_1, M_2), C = diag(C_1, C_2)$, and $u^\top = [u_1^\top, u_2^\top]$. Combining the equations of motion using the constraints (2.4) and (2.6), we obtain the dynamic equations representing the multiple manipulator system

$$(M + W^+ M_o W^{+\top})\ddot{z} + (C + W^+ C_o W^{+\top})\dot{z} + P f_{int} = u \qquad (3.4)$$

When the two manipulators rigidly grasp the object the position and orientation of the object can be entirely specified by the position and orientation of one of the manipulators [5]. Without loss of generality we can take z_1 as the independent set of coordinates and z_2 as the dependent coordinates and write $z_2 = \Omega(z_1)$. Differentiating this two times successively with respect to time, we obtain

$$\dot{z}_2 = T_1(z_1)\dot{z}_1, \qquad \ddot{z}_2 = T_1\ddot{z}_1 + \dot{T}_1\dot{z}_1 \qquad (3.5)$$

where $T_1 = \frac{\partial\Omega}{\partial z_1}$. By defining $T^\top = [I_6, T_1^\top]$, we can express the velocity and acceleration of z in terms of the independent coordinate z_1 as

$$\dot{z} = T\dot{z}_1, \qquad \ddot{z} = T\ddot{z}_1 + \dot{T}\dot{z}_1 \qquad (3.6)$$

Notice that from equation (2.5) the velocity $\dot{z}_2$ can be written in terms of $\dot{z}_1$ as follows,

$$\dot{z}_2 = W_2^\top W_1^{-\top}\dot{z}_1 \qquad (3.7)$$

This means that the transformation T_1 is $W_2^\top W_1^{-\top}$. So z_2 is a linear function of z_1 i.e., Ω is linear in z_1. Moreover T_1 is a constant transformation, which implies that $\dot{T}_1$ is zero. Notice that $T^\top = [I_6, W_1^{-1}W_2]$ and is known, since $W_2^\top W_1^{-\top}$ depends on the position of the contact points and not the center of mass of the object. We can also relate T and W as $T^\top = W_1^{-1}W$. Therefore the equations of motion (3.4) can be written in terms of the independent coordinate z_1 as

$$[(M + W^+ M_o W^{+\top})T]\ddot{z}_1 + [(C + W^+ C_o W^{+\top})T]\dot{z}_1 + P f_{int} = u \qquad (3.8)$$

4. Controller Design

The control u can be written as sum of two components, $u = u_m + u_f$, such that $W u_f = 0$ i.e., u_f lies in the null space of W. Notice that with $u_m^\top = [u_{m1}^\top, u_{m2}^\top]$ and $u_f^\top = [u_{f1}^\top, u_{f2}^\top]$, the control of individual manipulators are $u_1 = u_{m1} + u_{f1}$ and $u_2 = u_{m2} + u_{f2}$. Premultiplying the dynamic equation of the manipulator system with $T^\top$ we obtain,

$$H(z_1)\ddot{z}_1 + N(z_1, \dot{z}_1)\dot{z}_1 = v \qquad (4.1)$$

where $H = [T^\top M T + W_1^{-1} M_o W_1^{-\top}], N = [T^\top C T + W_1^{-1} C_o W_1^{-\top}]$ and $v = T^\top u = T^\top u_m$. We choose the following motion control law, internal force control law and the parameter adaptation law for the unknown object parameters as

$$v = \widehat{H}(z_1)\ddot{z}_1^d + \widehat{N}(z_1, \dot{z}_1)\dot{z}_1^d - (K_p + \lambda\widehat{N})e - K_d\dot{e} \qquad (4.2)$$

$$u_f = P(f_d - K_{fp}e_f - K_{fI}\int_{t_o}^{t} e_f(p)dp) \qquad (4.3)$$

$$\dot{\widehat{m}} = \gamma_m^{-1} e_v^\top \mathcal{A}_m(t) \tag{4.4}$$

$$\dot{\widehat{I}}_{kj} = \gamma_{kj}^{-1} e_v^\top \mathcal{A}_{kj}(t) \tag{4.5}$$

$$\dot{\widehat{\rho}}_{mj} = \gamma_{mj}^{-1} e_v^\top \mathcal{B}_{mj}(t) \tag{4.6}$$

$$\dot{\widehat{\xi}}_{kj} = \lambda_{kj}^{-1} e_v^\top \mathcal{D}_{kj}(t) \tag{4.7}$$

where $\widehat{H} = [T^\top MT + \widehat{W}_1^+ \widehat{M}_o \widehat{W}_1^{+\top}]$, $\widehat{N} = [T^\top CT + \widehat{W}_1^+ \widehat{C}_o \widehat{W}_1^{+\top}]$, z_1^d is the desired cartesian trajectory of the end effector, f_d is the desired internal force trajectory, $e = z_1 - z_1^d$ is the position error, $e_f = f_{int} - f_d$ is the internal force error, $e_v = \dot{e} + \lambda e$ is the reference velocity error. K_p and K_d are the positive definite position gain matrix and velocity gain matrix respectively, $\lambda, \gamma_m, \gamma_{kj}, \gamma_{mj}, \lambda_{kj}$ are positive constants and $\widehat{m}$ is the estimated mass of the object. $\widehat{I}_{kj}$ $(k, j = x, y, z)$ are the estimated nine inertia components of the object, $\widehat{\rho}_m$ is the product of the mass of the object and the position of the center of mass of the object with respect to contact point of manipulator 1 and $\widehat{\xi}_{ij}$ are the estimated parameters which are products of mass of the object and square of the position of center of mass of manipulator 1. The terms $\widehat{I}_{kj}$, $\widehat{\rho}_m$, and $\widehat{\xi}_{ij}$ are defined clearly in the appendix. The expressions for $\mathcal{A}_m, \mathcal{A}_{ij}, \mathcal{B}_m, \mathcal{D}_{kj}$ are derived in the appendix. Substituting the control in the equations of motion we obtain the error equation

$$\begin{aligned} H\ddot{e} + N\dot{e} = {} & (\widehat{H} - H)\ddot{z}_1^d + (\widehat{N} - N)\dot{z}_1^d \\ & - (K_p + \lambda N)e - K_d\dot{e} \end{aligned} \tag{4.8}$$

Theorem 4.1 *a) With the motion control law (4.2) and the parameter adaptation laws (4.4)-(4.7) and with proper choice of the gain matrices K_d and K_p the motion errors, e and $\dot{e}$ asymtotically converge to zero and the adaptation parameters are bounded.*

b) Under the force control law (4.3) the force error e_f is bounded.

Proof: *a)*

Consider the following Lyapunov function candidate

$$\begin{aligned} V(e, \dot{e}, \eta) = {} & \frac{1}{2} e_v^\top H e_v + \frac{1}{2} e^\top K_p e + \frac{1}{2}\gamma_m \widetilde{m}^2 \\ & + \frac{1}{2}\sum_{j=x}^{z} \gamma_{mj}\widetilde{\rho}_{mj}^2 + \frac{1}{2}\sum_{k=x}^{z}\sum_{j=x}^{z} \gamma_{kj}\widetilde{I}_{kj}^2 \\ & + \frac{1}{2}\sum_{k=x}^{z}\sum_{j=x}^{z} \lambda_{kj}\widetilde{\xi}_{kj}^2 \end{aligned} \tag{4.9}$$

where $e_v = \dot{e} + \lambda e$, $\widetilde{m} = \widehat{m} - m$, $\widetilde{I}_{kj} = \widehat{I}_{kj} - I_{kj}$, $\widetilde{\rho}_{mj} = \widehat{\rho}_{mj} - \rho_{mj}$, $\widetilde{\xi}_{kj} = \widehat{\xi}_{kj} - \xi_{kj}$ and η denotes the combined parameter error set of the object. Taking the time derivative of (4.9) along the trajectories of (4.8) we get

$$\begin{aligned} \dot{V}(e, \dot{e}) = {} & e_v^\top H \dot{e}_v + \frac{1}{2} e_v^\top \dot{H} e_v + \dot{e}^\top K_p e + \gamma_m \widetilde{m}\dot{\widetilde{m}} \\ & + \sum_{j=x}^{z} \gamma_{mj}\dot{\widetilde{\rho}}_{mj}\widetilde{\rho}_{mj} + \sum_{k=x}^{z}\sum_{j=x}^{z} \gamma_{kj}\dot{\widetilde{I}}_{kj}\widetilde{I}_{kj} \\ & + \sum_{k=x}^{z}\sum_{j=x}^{z} \lambda_{kj}\dot{\widetilde{\xi}}_{kj}\widetilde{\xi}_{kj} \end{aligned} \tag{4.10}$$

Using the skew symmetry property of $\dot{H} - 2N$, error equation (4.8) , and grouping the terms together

$$\begin{aligned} \dot{V}(e, \dot{e}) = {} & -e_v^\top [K_d - \lambda H]\dot{e} - \lambda e^\top K_p e + e_v^\top [(\widehat{H} - H)\ddot{z}^d \\ & + (\widehat{N} - N)\dot{z}^d + \gamma_m \widetilde{m}\dot{\widetilde{m}} + \sum_{j=x}^{z} \gamma_{mj}\dot{\widetilde{\rho}}_{mj}\widetilde{\rho}_{mj} \\ & + \sum_{k=x}^{z}\sum_{j=x}^{z} \gamma_{kj}\dot{\widetilde{I}}_{kj}\widetilde{I}_{kj} + \sum_{k=x}^{z}\sum_{j=x}^{z} \lambda_{kj}\dot{\widetilde{\xi}}_{kj}\widetilde{\xi}_{kj} \end{aligned} \tag{4.11}$$

With the parameter adaptation laws (4.4)-(4.7) defined previously, we obtain

$$\begin{aligned} \dot{V}(e, \dot{e}) = {} & -(\dot{e} + \frac{\lambda}{2}e)^\top [K_d - \lambda H](\dot{e} + \frac{\lambda}{2}e) \\ & - \frac{\lambda}{4} e^\top [4K_p + \lambda^2 H - \lambda K_d] e \end{aligned} \tag{4.12}$$

$\dot{V}(e, \dot{e})$ is negative semi-definite by choosing the positive definite gain matrics K_d and K_p such that $\lambda_{min}[K_d - \lambda H] \geq 0$ and $\lambda_{min}[4K_p + \lambda^2 H - \lambda K_d] \geq 0$, where $\lambda_{min}(A)$ is the smallest eigenvalue of matrix A. Since V is positive definite, from the above analysis we can conclude that $e, e_v \in L_\infty \cap L_2$. Also from the error equation (4.8) $\ddot{e} \in L_\infty$. Thus, e and $\dot{e}$ converge to zero.

◇

Proof: *b)*

To obtain the force error equation and for implementation we need to have u_{m1} and u_{m2}. There are many ways to choose u_{m1} and u_{m2} from the control v. Later we will discuss one way of obtaining these from the control law v. The force error equation can be obtained by substitution of the force control law (4.3) and the motion controls u_{m1} and u_{m2} into the dynamic equation (3.8) as

$$P(e_f + K_{fp}e_f + K_{fI}\int_0^t e_f(p)dp) = \beta(e, \dot{e}, \ddot{e}, \eta) \tag{4.13}$$

where η is the unknown parameter set of the object. Since the right side of (4.13) is bounded, exactly the same analysis as done in [5] shows that the force error e_f is bounded. Moreover if $\beta(e, \dot{e}, \ddot{e}, \eta)$ converges to zero, then the force error e_f converges to zero. ◇

Now we show one way of obtaining u_{m1} and u_{m2} from the control law v. The control v is a function of both motion torques u_{m1} and u_{m2} as

$$v = u_{m1} + \widehat{W}_1^{-1}\widehat{W}_2 u_{m2} \tag{4.14}$$

We obtain a relationship between u_{m1} and u_{m2} by minimizing a cost function which is quadratic in u_{m1} and u_{m2},

$$J_u = \frac{1}{2}(u_{m1}^\top u_{m1} + u_{m2}^\top u_{m2}) \tag{4.15}$$

Minimizing J_u with respect to u_{m1} and equating this to zero we obtain,

$$\frac{\partial J_u}{\partial u_{m1}} = u_{m1} + \frac{\partial u_{m2}}{\partial u_{m1}} u_{m2} = 0 \tag{4.16}$$

Another relationship between u_1 and u_2 is obtained from the dynamic equation as

$$u_{m1} + \widehat{W}_1^{-1}\widehat{W}_2 u_{m2} = \alpha(z_1, \dot{z}_1, \ddot{z}_1) \tag{4.17}$$

The right side of this equation is a function of $z_1, \dot{z}_1, \ddot{z}_1$ and the inertial parameters of the manipulators and the object, all of which are bounded. Also notice that the right side of this equation does not include terms in u_{m1} and u_{m2}. Taking the partial of this equation with respect u_{m1} we obtain $\frac{\partial u_{m2}}{\partial u_{m1}} = -\widehat{W}_2^{-1}\widehat{W}_1$. Substituting this expression in (4.16), we obtain the relationship between u_{m1} and u_{m2} as

$$\widehat{W}_2 u_{m1} = \widehat{W}_1 u_{m2} \tag{4.18}$$

Using this equation and equation (4.14) we obtain individual expressions for torques u_{m1} and u_{m2} as

$$u_{m1} = [\widehat{W}_1\widehat{W}_2^{-1} + \widehat{W}_2\widehat{W}_1^{-1}]^{-1}\widehat{W}_2^{-1} v \tag{4.19}$$

$$u_{m2} = [\widehat{W}_1\widehat{W}_2^{-1} + \widehat{W}_2\widehat{W}_1^{-1}]^{-1}\widehat{W}_2^{-1} v \tag{4.20}$$

5. Conclusions

An adaptive controller has been developed for two robot manipulators carrying an unknown object. The object was assumed to be completely unknown, i.e., uncertainties in the inertial parameters (mass and inertia) and also the center of mass of the object were considered. An adaptive motion control law and a feedforward plus PI type internal force control law are developed to track the desired motion trajectory and internal force trajectory of the object. Parameter adaptation laws for estimating the unknown parameters of the object are derived, although some overparametrization has been done to achieve this. To specify the individual torques of the manipulators an optimal control problem was formulated by minimizing a cost function.

Appendix

We define the following matrices

$$E_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad E_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$E_z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{xx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_{yy} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{zz} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{xy} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{xz} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$E_{yz} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The matrices $\widetilde{H} := \widehat{H} - H$ and $\widetilde{N} := \widehat{N} - N$ can be written in the following manner for seperation of the unknown parameters,

$$\begin{aligned} \widetilde{H} &= \widehat{W}_1^{-1}\widehat{M}_o\widehat{W}_1^{-\top} - W_1^{-1}M_oW_1^{-\top} \\ \widetilde{N} &= \widehat{W}_1^{-1}\widehat{C}_o\widehat{W}_1^{-\top} - W_1^{-1}C_oW_1^{-\top} \end{aligned}$$

$$\begin{aligned} \widetilde{H}\ddot{z}_1^d + \widetilde{N}\dot{z}_1^d = {} & [\widehat{W}_1^{-1}\widehat{M}_o\widehat{W}_1^{-\top} - W_1^{-1}M_oW_1^{-\top}]\ddot{z}_1^d \\ & + [\widehat{W}_1^{-1}\widehat{C}_o\widehat{W}_1^{-\top} - W_1^{-1}C_oW_1^{-\top}]\dot{z}_1^d \end{aligned} \tag{5.1}$$

$$\begin{aligned} W_1^{-1}M_oW_1^{-\top} &= \begin{bmatrix} I_3 & O_3 \\ -Q_1 & I_3 \end{bmatrix} \begin{bmatrix} mI_3 & O_3 \\ O_3 & I_o \end{bmatrix} \begin{bmatrix} I_3 & Q_1 \\ O_3 & I_3 \end{bmatrix} \\ &= \begin{bmatrix} mI_3 & mQ_1 \\ -mQ_1 & I_o - mQ_1^2 \end{bmatrix} \end{aligned} \tag{5.2}$$

$$W_1^{-1} C_o W_1^{-\top} = \begin{bmatrix} I_3 & O_3 \\ -Q_1 & I_3 \end{bmatrix} \begin{bmatrix} O_3 & O_3 \\ O_3 & S_o I_o \end{bmatrix} \begin{bmatrix} I_3 & Q_1 \\ O_3 & I_3 \end{bmatrix} = \begin{bmatrix} O_3 & O_3 \\ O_3 & S_o I_o \end{bmatrix} \tag{5.3}$$

where $S_o = [\omega_1 \times]$ and I_o is the inertia matrix of the object. Since Q_1 is a 3×3 skew symmetric matrix, Q_1^2 is a symmetric negative definite matrix. Notice that the matrix Q_1^2 is

$$\begin{bmatrix} -(\rho_{1y}^2 + \rho_{1z}^2) & \rho_{1x}\rho_{1y} & \rho_{1z}\rho_{1x} \\ \rho_{1x}\rho_{1y} & -(\rho_{1z}^2 + \rho_{1x}^2) & \rho_{1y}\rho_{1z} \\ \rho_{1z}\rho_{1x} & \rho_{1y}\rho_{1z} & -(\rho_{1x}^2 + \rho_{1y}^2) \end{bmatrix}$$

To obtain a parametrization for mQ_1 and mQ_1^2 we define the following terms

$$\begin{gathered} \rho_{mx} := m\rho_{1x} \quad \rho_{my} := m\rho_{1y} \quad \rho_{mz} := m\rho_{1z} \\ \xi_{xx} := -m(\rho_{1y}^2 + \rho_{1z}^2) \quad \xi_{yy} := -m(\rho_{1z}^2 + \rho_{1x}^2) \\ \xi_{zz} := -m(\rho_{1x}^2 + \rho_{1y}^2) \quad \xi_{xy} := \rho_{1x}\rho_{1y} \\ \xi_{yz} := m\rho_{1z}\rho_{1y} \quad \xi_{zx} := m\rho_{1z}\rho_{1x} \end{gathered}$$

Now we define the parameter errors as $\tilde{\rho}_{mj} := \hat{\rho}_{mj} - \rho_{mj}$ $(j = x, y, z)$ and $\tilde{\xi}_{kj} := \hat{\xi}_{kj} - \xi_{kj}$ $(k, j = x, y, z)$. With these definitions the matrices $\widetilde{I}_o$, mQ_1, and mQ_1^2 can be expressed in terms of the elementary matrices defined earlier and the unknown object parameters as

$$\begin{aligned} \widetilde{I}_o = {} & \widetilde{I}_{xx} E_{xx} + \widetilde{I}_{yy} E_{yy} + \widetilde{I}_{zz} E_{zz} \\ & + \widetilde{I}_{xy} E_{xy} + \widetilde{I}_{yz} E_{yz} + \widetilde{I}_{zx} E_{zx} \end{aligned} \tag{5.4}$$

$$\begin{aligned} \widehat{m}\widehat{Q}_1^2 - mQ_1^2 = {} & \widetilde{\xi}_{xx} E_{xx} + \widetilde{\xi}_{yy} E_{yy} + \widetilde{\xi}_{zz} E_{zz} \\ & + \widetilde{\xi}_{xy} E_{xy} + \widetilde{\xi}_{yz} E_{yz} + \widetilde{\xi}_{zx} E_{zx} \end{aligned} \tag{5.5}$$

$$\widehat{m}\widehat{Q}_1 - mQ_1 = \widetilde{\rho}_{mx} E_x + \widetilde{\rho}_{my} E_y + \widetilde{\rho}_{mz} E_z \tag{5.6}$$

Now we can write the expressions for $\mathcal{A}_m, \mathcal{A}_{kj}, \mathcal{B}_j$, and $\mathcal{D}_{kj}$ as

$$\mathcal{A}_m = \begin{bmatrix} \widetilde{m} I_3 & O_3 \\ O_3 & O_3 \end{bmatrix} \ddot{z}_1^d \tag{5.7}$$

$$\mathcal{A}_{kj} = \begin{bmatrix} O_3 & O_3 \\ O_3 & E_{kj} \end{bmatrix} \ddot{z}_1^d + \begin{bmatrix} O_3 & O_3 \\ O_3 & S_o E_{kj} \end{bmatrix} (\dot{z}_1^d - \lambda e) \tag{5.8}$$

$$\mathcal{B}_{mj} = \begin{bmatrix} O_3 & E_j \\ -E_j & O_3 \end{bmatrix} \ddot{z}_1^d \tag{5.9}$$

$$\mathcal{D}_{kj} = \begin{bmatrix} O_3 & O_3 \\ O_3 & E_{kj} \end{bmatrix} \ddot{z}_1^d \tag{5.10}$$

where $j = x, y, z$ and $k = x, y, z$.

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