Semi-Globally Stable Decentralized Control of a Class of Large-Scale Interconnected Nonlinear Systems[1]

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Abstract: Decentralized control of a class of large-scale interconnected nonlinear systems with uncertain and bounded nonlinear interconnections is considered in this paper. Both state regulation and state tracking problems are considered. In the case of state regulation it is shown that a linear decentralized feedback controller achieves semi-global exponential stability. In the case of state tracking, a new decentralized discontinuous control law is designed and is shown to achieve semi-global exponential stability; a decentralized continuous control law is also designed, which guarantees that each subsystem tracking error is ultimately bounded within a certain neighborhood of zero. For both the regulation and the tracking cases, the proposed controllers are compared with an existing higher-order global controller via simulation examples.

1 Introduction

During the past two decades there has been a strong interest in the study of decentralized control of large-scale interconnected nonlinear systems. An important motivation for design of decentralized schemes is that the information exchange between subsystems of a large-scale system is not needed. A large body of literature on decentralized control of large-scale systems can be found in [1]. The focus of much of the early research was on decentralized control of large-scale systems with first-order interconnections [2, 3]. Recently, an LMI scheme for design of autonomous decentralized controllers and observers for output control of complex systems has been proposed in [4] for a class of systems whose interconnections satisfy quadratic constraints in the states.

In [5], global decentralized schemes were proposed for large-scale systems with higher-order interconnections; the interconnections were assumed to be bounded by a p-th order polynomial in states. Decentralized adaptive control of large-scale interconnected systems in strict feedback form with interconnections bounded by p-th order polynomials in states can be found in [6]. In both of these papers the resulting decentralized controller utilizes $(2p-1)$-th order in states to achieve global stability. Due to the use of the $(2p-1)$-th order terms in the controller to compensate for the p-th order terms, the control effort that is required is very large; further, these controllers require large variations in the control effort in a short duration of time, especially when the initial condition errors are large. In [7] a non-smooth variable structure controller is designed for regulation of large-scale systems with higher-order interconnections. Although convergence of the system trajectories to the sliding surface of each individual subsystems is shown by using a p-th order controller, the convergence of the regulation error to zero once it is on the sliding surface is not shown. Decentralized adaptive control of large-scale systems with both linear and nonlinear interconnections is considered in [8]; it is shown that asymptotic tracking of desired states with zero error is possible for each subsystem, under the assumption that each subsystem has knowledge of only the desired states of other subsystems. In [9] a decentralized adaptive controller has been proposed for a class of large-scale systems that are in special output-feedback normal form; the proposed decentralized controllers are totally decentralized for the set-point regulation problem and are partially decentralized for the tracking problem.

In this paper, we consider a class of large-scale interconnected nonlinear systems with higher-order interconnections, which are assumed to be bounded by p-th order polynomial in states. Both the regulation and the tracking cases are considered. In the case of regulation, semi-global exponential stability is shown with a linear decentralized controller; that is, the states converge to zero exponentially and the region of attraction can be increased arbitrarily by choosing the control gain. In the case of tracking, a new decentralized controller is proposed for two different types of nonlinear interconnections; one assumes that the nonlinear interconnections are known and the other assumes that the interconnections are just bounded by higher-order polynomials in states. For the tracking case, the proposed decentralized controllers assume availability of the desired states of each subsystem to all individual subsystems.

The remainder of the paper is organized as follows. Section 2 gives a description of the class of large-scale interconnected system considered in this paper. The regulation and the tracking cases are considered in Section 3 and Section 4, respectively. Comparative simulation results are also presented and discussed on some illustrative examples for both regulation and tracking in Section 5. Conclusions of this paper and future research are given in Section 6.

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2 Description and Preliminaries

Consider the class of large-scale systems consisting of the following interconnected subsystems:

$$\dot{z}_i(t) = A_i z_i(t) + b_i u_i(t) + f_i^i(z(t)) \tag{1}$$

where $i = 1, \ldots, N$, $z_i(t) \in \mathbb{R}^{n_i}$ and $u_i(t) \in \mathbb{R}$ denote the state and input, respectively, of the i-th subsystem, $z(t) \in \mathbb{R}^{n_1+\cdots+n_N}$ denotes the state of the overall system, $A_i \in \mathbb{R}^{n_i \times n_i}$, and $b_i \in \mathbb{R}^{n_i}$.

Assumptions: (I) The pairs (A_i, b_i) are controllable. The vector fields $f_i^i(x_1(t), x_2(t), \ldots, x_N(t)) \in \mathbb{R}^{n_i}$ are smooth with $f_i^i(0, 0, \ldots, 0) = 0$. (II) The interconnections satisfy the matching condition; that is, $f_i^i(z(t)) = b_i w_i(z(t))$; and $w_i(z)$ is polynomially bounded as follows:

$$|w_i(z(t))| \le \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|z_j(t)\|^k. \tag{2}$$

(III) The state of each individual subsystem is available to its decentralized controller. For the tracking case, the desired state of all the subsystems is available to the decentralized controller of each subsystem.

The following three algebraic inequalities will be used in the stability proofs of the regulation and the tracking control problems. For any $z_i \in \mathbb{R}^{n_i}$, $z_j \in \mathbb{R}^{n_j}$, and $a_i \in \mathbb{R}_+$ and any positive integers p, k, and N, the following inequalities are true:

$$\|z_i\|\|z_j\|^p + \|z_i\|^p\|z_j\| \le \|z_i\|^{p+1} + \|z_j\|^{p+1}, \tag{3}$$

$$\sum_{i=1}^{N} \|z_i\| \sum_{j=1}^{N} \|z_j\|^k \le N \sum_{i=1}^{N} \|z_i\|^{k+1}, \tag{4}$$

$$(a_1 + a_2 + \cdots + a_N)^k \le N^{k-1}(a_1^k + a_2^k + \cdots + a_N^k). \tag{5}$$

3 State Regulation

In this section it will be shown that a linear decentralized controller for each subsystem render the equilibria, $z_i(t) = 0$, $i = 1 : N$, semi-globally exponential stable. In the following we will assume that i and j take values from 1 to N. Consider the following control input:

$$u_i = -k_i^T z_i. \tag{6}$$

where k_i is the feedback gain vector. The closed-loop dynamics with the linear controller is given by

$$\dot{z}_i = (A_i - b_i k_i^T) z_i + b_i w_i(z). \tag{7}$$

Since the pair (A_i, b_i) is controllable, the eigenvalues of $\bar{A}_i := (A_i - b_i k_i^T)$ can be arbitrarily assigned by choosing the feedback gain vector k_i. Therefore, for any positive definite matrix Q_i there exists a unique positive definite matrix P_i such that the following Lyapunov equation is satisfied:

$$\bar{A}_i^T P_i + P_i \bar{A}_i = -Q_i. \tag{8}$$

Let σ_{qi} be the minimum eigenvalue of Q_i, $\sigma_{1i} := \|P_i b_i\|$, $\sigma_{2i} := \max_{j,k} \beta_{ijk}$, and $p_{mi} := \max_j p_{ij}$.

Theorem 3.1 *Consider the large-scale interconnected nonlinear system* (1) *that satisfies the assumptions (I) through (III); and let the control input be given by* (6). *If* $\|z_i(t_0)\| \le R_{ri}$ *and the feedback gain vector is chosen such that*

$$\frac{\sigma_{qi}}{\sigma_{1i}} > 2N\sigma_{2i} \sum_{k=1}^{p_{mi}} R_{ri}^{k-1}, \tag{9}$$

then the equilibria $z_i = 0$ *are semi-globally exponentially stable; that is, the region of attraction for each subsystem,* R_{ri}*, can be increased arbitrarily by choosing the corresponding feedback gain vector* k_i*.*

Proof. Consider the following Lyapunov function candidate for the interconnected system:

$$V(z) = \sum_{i=1}^{N} z_i^T P_i z_i. \tag{10}$$

The time derivative of the Lyapunov function candidate along the trajectories of the closed-loop system (7) satisfies

$$\begin{aligned}
\dot{V}(z) &= \sum_{i=1}^{N} -z_i^T Q_i z_i + 2 z_i^T P_i b_i w_i \\
&\le -\sum_{i=1}^{N} z_i^T Q_i z_i + \sum_{i=1}^{N} 2 z_i^T P_i b_i \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|z_j\|^k \\
&\le -\sum_{i=1}^{N} z_i^T Q_i z_i + \sum_{i=1}^{N} 2\|P_i b_i\|\|z_i\| \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|z_j\|^k \\
&\le -\sum_{i=1}^{N} z_i^T Q_i z_i + \sum_{i=1}^{N} 2\sigma_{1i}\sigma_{2i}\|z_i\| \sum_{j=1}^{N} \sum_{k=1}^{p_{mi}} \|z_j\|^k.
\end{aligned} \tag{11}$$

Using the inequality (4) repeatedly, the derivative of the Lyapunov function candidate satisfies

$$\begin{aligned}
\dot{V}(z) &\le -\sum_{i=1}^{N} z_i^T Q_i z_i + 2N \sum_{i=1}^{N} \sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|z_i\|^{k+1} \\
&\le -\sum_{i=1}^{N} \left(\sigma_{qi}\|z_i\|^2 - 2N\sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|z_i\|^{k+1} \right).
\end{aligned} \tag{12}$$

If $p_{mi} = 1$, then the linear controller will result in global exponential stability via the choice of the feedback gain vector k_i. Assume $p_{mi} \ge 2$, then

$$\begin{aligned}
\dot{V}(z) &\le -\sum_{i=1}^{N} \|z_i\|^2 \left(\sigma_{qi} - 2N\sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|z_i\|^{k-1} \right) \\
&\le -\sum_{i=1}^{N} \gamma_i \|z_i\|^2
\end{aligned} \tag{13}$$

where the second inequality is obtained by using (9) and γ_i is some positive number. This concludes the proof of Theorem 3.1.

Remark 3.1 *Since N and σ_{2i} are fixed, the region of attraction can be made larger by increasing the ratio σ_{qi}/σ_{1i}. Notice that σ_{qi} is the minimum eigenvalue of Q_i and $\sigma_{1i} = \|P_i b_i\|$. Therefore one can increase the ratio by appropriately placing the poles of the i-th closed-loop system matrix, $(A_i - b_i k_i^T)$, by choosing the gain vector k_i.*

Remark 3.2 *Notice that that there is no communication between individual subsystems. Further, the region of attraction for each subsystem can be increased by the choice of the feedback gain vector of that subsystem alone.*

4 State Tracking

Let the N reference models for the N subsystems be described by

$$\dot{z}_{mi}(t) = A_{mi} z_{mi}(t) + b_i r_i(t) \tag{14}$$

where $r_i(t)$ are bounded reference inputs and $z_{mi}(t)$ are the reference trajectories and $A_{mi} := A_i - b_i k_i^T$ are asymptotically stable matrices by the choice of the vectors k_i. Hence, for any positive definite Q_i there exists a unique positive definite P_i such that the following Lyapunov equations are satisfied:

$$A_{mi}^T P_i + P_i A_{mi} = -Q_i. \tag{15}$$

The goal is to find $u_i(t)$ such that $z_i(t)$ are bounded and

$$\lim_{t\to\infty} \|e_i(t)\| := \lim_{t\to\infty} \|z_i(t) - z_{mi}(t)\| = 0, \ i = 1, 2, \ldots, N.$$

Choose the control input $u_i(t)$ as follows:

$$u_i(t) = r_i(t) - v_i(z_m(t)) - k_i^T z_i(t) \tag{16}$$

where $v_i(z_m(t))$ depends on the reference state and will be given later. The error dynamics resulting from the above controllers are:

$$\dot{e}_i(t) = A_{mi} e_i(t) + b_i(w_i(z(t)) - v_i(z_m(t))). \tag{17}$$

We consider the following two cases for the nonlinear interconnections $w_i(z(t))$ and choose $v_i(z_m)$ for each case:

(1) The structure of $w_i(z(t))$ is known and satisfies

$$|w_i(z(t)) - w_i(z_m(t))| \le \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|e_j(t)\|^k. \tag{18}$$

For this case $v_i(z_m(t))$ is chosen as

$$v_i(z_m(t)) = w_i(z_m(t)). \tag{19}$$

(2) Only a bound on $w_i(z(t))$ is known and is given by (2), which is reproduced below:

$$|w_i(z(t))| \le \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|z_j(t)\|^k. \tag{20}$$

For this case $v_i(z_m(t))$ is chosen as

$$v_i(z_m(t)) = \begin{cases} \dfrac{e_i^T P_i b_i}{|e_i^T P_i b_i|}\, \rho_i(z_m) & \text{if } |e_i^T P_i b_i| \neq 0, \\ 0 & \text{if } |e_i^T P_i b_i| = 0 \end{cases} \tag{21}$$

where

$$\rho_i(z_m) := \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} 2^{k-1} \beta_{ijk} \|z_{mj}(t)\|^k. \tag{22}$$

Notice that case **(2)** is much more general than case **(1)**. Further, notice that in both cases $v_i(z_m(t))$ is a function of the desired states of all the subsystems, which are assumed to be known a priori and hence available to each subsystem controller. As in the regulation case, let σ_{qi} be the minimum eigenvalue of Q_i, $\sigma_{1i} := \|P_i b_i\|$, $\sigma_{2i} := \max_{j,k} \beta_{ijk}$, and $p_{mi} := \max_j p_{ij}$. The following theorem gives stability of the tracking controller.

Theorem 4.1 *Consider the large-scale interconnected nonlinear system* (1) *that satisfies assumptions (I) through (III); and let the control input be given by* (16). *If $\|e_i(t_0)\| \le R_{ti}$ and the feedback gain vector is chosen such that*

$$\frac{\sigma_{qi}}{\sigma_{1i}} > 2^{p_{mi}} N \sigma_{2i} \sum_{k=1}^{p_{mi}} R_{ti}^{k-1}, \tag{23}$$

then the tracking errors $e_i(t)$ exponentially converge to zero; further, the region of attraction R_{ti} can be increased arbitrarily by choosing the corresponding feedback gain vector k_i.

Proof. Consider the following Lyapunov function candidate:

$$V(e) = \sum_{i=1}^{N} e_i^T P_i e_i. \tag{24}$$

Taking the time derivative and using the error dynamics results in

$$\dot{V}(e) = \sum_{i=1}^{N} -e_i^T Q_i e_i + 2 e_i^T P_i b_i (w_i(z) - v_i(z_m)). \tag{25}$$

We consider the two cases as given above:

(1) Substituting (19) into (25) and simplifying using the bound (18) gives

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} -e_i^T Q_i e_i + 2 e_i^T P_i b_i (w_i(z) - w_i(z_m)) \\ &\le \sum_{i=1}^{N} -e_i^T Q_i e_i + 2 |e_i^T P_i b_i| |w_i(z) - w_i(z_m)| \\ &\le \sum_{i=1}^{N} -e_i^T Q_i e_i + \sum_{i=1}^{N} 2 |e_i^T P_i b_i| \sum_{j=1}^{N} \sum_{k=1}^{p_{ij}} \beta_{ijk} \|e_j(t)\|^k. \end{aligned}$$

(2) Noting that $z_j = z_{mj} + e_j$ and using the bound on $w_i(z)$

$$\begin{aligned}\dot V &= \sum_{i=1}^{N} -e_i^T Q_i e_i + 2e_i^T P_i b_i (w_i(z_m+e) - v_i(z_m)) \\ &\leq \sum_{i=1}^{N} -e_i^T Q_i e_i - 2e_i^T P_i b_i v_i(z_m) \\ &\qquad + 2|e_i^T P_i b_i| \sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} \beta_{ijk} \|z_{mj} + e_j\|^k \\ &\leq \sum_{i=1}^{N} -e_i^T Q_i e_i - 2e_i^T P_i b_i v_i(z_m) \\ &\qquad + 2|e_i^T P_i b_i| \sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} \beta_{ijk} (\|z_{mj}\| + \|e_j\|)^k.\end{aligned}$$

Using (21) and the inequality $(\|z_{mj}\| + \|e_j\|)^k \leq 2^{k-1}(\|z_{mj}\|^k + \|e_j\|^k)$, $k \geq 2$, we obtain

$$\begin{aligned}\dot V(e) &\leq -\sum_{i=1}^{N} e_i^T Q_i e_i \\ &\qquad + \sum_{i=1}^{N} 2|e_i^T P_i b_i| \sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} \beta_{ijk} 2^{k-1} \|e_j\|^k \\ &\leq -\sum_{i=1}^{N} e_i^T Q_i e_i \\ &\qquad + \sum_{i=1}^{N} 2|e_i^T P_i b_i| \sum_{j=1}^{N}\sum_{k=1}^{p_{ij}} \beta_{ijk} 2^{k-1} \|e_j\|^k.\end{aligned}$$

Therefore, in both cases the derivative of the Lyapunov function candidate satisfies

$$\begin{aligned}\dot V(e) &\leq -\sum_{i=1}^{N} e_i^T Q_i e_i \\ &\qquad + \sum_{i=1}^{N} 2\sigma_{1i}\sigma_{2i} 2^{p_{mi}-1} \|e_i\| \sum_{j=1}^{N}\sum_{k=1}^{p_{mi}} \|e_j\|^{k+1}.\end{aligned}$$

Repeatedly using claim 2, the derivative of the Lyapunov function candidate satisfies

$$\dot V(e) \leq \sum_{i=1}^{N} -e_i^T Q_i e_i + N \sum_{i=1}^{N} 2^{p_{mi}} \sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|e_i\|^{k+1}. \tag{26}$$

Following along the same lines as done in the regulation case, we obtain

$$\begin{aligned}\dot V(e) &\leq -\sum_{i=1}^{N} \|e_i\|^2 \left(\sigma_{qi} - N 2^{p_{mi}} \sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|e_i\|^{k-1}\right) \\ &\leq -\sum_{i=1}^{N} \gamma_i \|z_i\|^2 \end{aligned} \tag{27}$$

where the second inequality is obtained by using (23) and γ_i is a positive constant. This concludes the proof of Theorem 4.1.

The controller for case **(2)** is not continuous and will result in chattering. A continuous control law with a boundary layer can be used instead [10]. The auxiliary input $v_i(z_m(t))$ given by (21) is changed to the following:

$$v_i(z_m(t)) = \begin{cases} \dfrac{e_i^T P_i b_i}{|e_i^T P_i b_i|}\, \rho_i(z_m(t)) & \text{if } |e_i^T P_i b_i| \geq \epsilon_i, \\ \dfrac{e_i^T P_i b_i}{\epsilon_i}\, \rho_i(z_m(t)) & \text{if } |e_i^T P_i b_i| < \epsilon_i \end{cases} \tag{28}$$

where ϵ_i is the tunable boundary layer thickness of subsystem i. In this case the derivative of the Lyapunov function candidate satisfies the following:

$$\begin{aligned}\dot V(e) &\leq -\sum_{i=1}^{N} \|e_i\|^2 \left(\sigma_{qi} - N 2^{p_{mi}} \sigma_{1i}\sigma_{2i} \sum_{k=1}^{p_{mi}} \|e_i\|^{k-1}\right) \\ &\qquad + \sum_{i=1}^{N} \frac{\epsilon_i}{2} \rho_i(z_m).\end{aligned} \tag{29}$$

As a result of choosing the continuous control laws we must change the sufficient condition given by (23) to the following:

$$\frac{\sigma_{qi}}{\sigma_{1i}} \geq \gamma_i + 2^{p_{mi}} N \sigma_{2i} \sum_{k=1}^{p_{mi}} R_{ti}^{k-1}. \tag{30}$$

where γ_i is a positive number. Therefore,

$$\dot V(e) \leq -\sum_{i=1}^{N} \gamma_i \sigma_{1i} \|e_i\|^2 + \sum_{i=1}^{N} \frac{\epsilon_i}{2} \rho_i(z_m). \tag{31}$$

$\dot V < 0$ if

$$\|e_i\| \geq \delta_i, \;\; \delta_i := \frac{\epsilon_i \rho_i(z_m)}{2\gamma_i \sigma_{1i}}. \tag{32}$$

Let S_i denote the smallest level surface of $V_i := e_i^T P_i e_i$ containing the ball $B(\delta_i)$ of radius δ_i centered at $e_i = 0$. If $e_i(t_0) \in S_i$ then the solution $e_i(t)$ remains in S_i. If $e_i(t_0) \notin S_i$ then V_i is decreasing along the solutions of (17) and the solution reaches the boundary of S_i in finite time. In fact, the solution reaches the boundary of S_i in time

$$\bar t_i = t_0 + \frac{\alpha_{i0} - \alpha_i}{c_{i0}} \tag{33}$$

where $\alpha_{i0} = e_i^T(t_0) P_i e_i(t_0)$, $c_{i0} = \min\{\gamma_i \sigma_{1i} \|e_i\|^2 - \frac{\epsilon_i}{2}\rho_i(z_m)\}$, and α_i is such that $\alpha_i = e_i^T P_i e_i$ defines the level surface S_i. See [10] for details.

5 A comparative simulation study

5.1 The regulation case

Consider the interconnected system composed of the following two subsystems.

$$\dot z_i = A_i z_i + b_i u_i + b_i w_i(z), \;\; i = 1, 2 \tag{34}$$

where $A_i = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $w_1(z) = z_{11}z_{21} + z_{21}^2, w_2(z) = z_{12}z_{21} + z_{11}^2$. The nonlinear interconnections $w_1(z)$ and $w_2(z)$ can be bounded, similar to (2) with $N = 2$ and $p_{ij} = 2$, as follows:

$$\begin{aligned} \|w_1(z)\| &\leq \beta_{11}\|z_1\|^2 + \beta_{12}\|z_2\|^2 \\ \|w_2(z)\| &\leq \beta_{21}\|z_1\|^2 + \beta_{22}\|z_2\|^2 \end{aligned} \tag{35}$$

Higher-order global controller: We first design the global decentralized control laws given in [5, 6], assuming that the coefficients of the polynomial bounds are known. The decentralized control laws for the subsystems are:

$$u_i = -\alpha_i B_i P_i z_i (1 + \|z_i\|^{2(p-1)}),\ i = 1, 2. \tag{36}$$

Since $p = 2$ for the example considered, notice that the control scheme uses cubic terms in the states to compensate for interconnections that are bounded by second-order terms. Global stability is shown by using the following Lyapunov function candidate

$$V(z) = \sum_{i=1}^{N} \sum_{k=1}^{p} (z_i^T P_i z_i)^k \tag{37}$$

where $p = \max_j p_{ij} = 2$ denotes the highest degree in the polynomial bound and P_i is the positive definite solution of the algebraic Ricatti equation $A_i^T P_i + P_i A_i - 2\alpha_i P_i B_i B_i^T P_i + Q_i = 0$. Simulation results with initial conditions $z_1^T(0) = [2, -2]$ and $z_2^T(0) = [3, -3]$ are shown in Figure 1; $\alpha_i = 5$ and $Q_i = diag(4, 4)$ is chosen to solve the above Ricatti equation. The top plot in Figure 1 shows the regulation of subsystem states to zero and the bottom plot shows the two subsystem control inputs; notice that the control input in the transient stage is very large.

Linear decentralized semi-global controller: The linear decentralized controllers given by (6) is simulated for the same plant with the same initial conditions as above. The region of attraction is chosen to be $R_r i = 5, i = 1, 2$; notice that $\|z_i(0)\| \leq R_{ri}$ for both subsystems. The right-hand-side of (9), which is $N\sigma_{2i}\sum_{k=1}^{p_{mi}} R_{ri}^{k-1}$, takes the value of 12. Thus, one needs $\sigma_{qi}/\sigma_{1i} > 12,\ i = 1, 2$. The choice of $k_i = [10 \quad 10]^T,\ \ i = 1, 2$ and Q_i equal to the identity matrix results in $\sigma_{qi}/\sigma_{1i} = 13.459,\ i = 1, 2$, which satisfies the sufficient condition. Simulation results using the linear decentralized controller are shown in Figure 2. Comparison of the simulation results with the two controllers, Figures 1 and 2, shows that the higher-order global controller requires a much larger control input as well as control input variation in the transient stage.

5.2 The tracking case

Consider the same large-scale system with nonlinear interconnections given by (34). Choose the reference models to be:

$$\dot{z}_{mi}(t) = A_{mi} z_{mi}(t) + b_i r_i(t) \tag{38}$$

where $A_{mi} = A_i - b_i k_i^T$. The reference signals are chosen to be $r_1(t) = 20\sin(2t)$ and $r_2(t) = 30\sin(t)$. The tracking error dynamics is given by

$$\dot{e}_i(t) = A_i e_i(t) + b_i u_i(t) + b_i w_i(z_i). \tag{39}$$

As in the case of regulation, we design the higher-order global controller first and compare it with the proposed low-order tracking controller. The decentralized global higher-order control laws are given by:

$$u_i(t) = r_i(t) - k_i z_{mi}(t) - \alpha_i b_i P_i e_i(t)(1 + \|e_i(t)\|^{2(p-1)}) \tag{40}$$

where $p = p_{i,j} = 2$ denotes the highest degree in the polynomial bound. The simulation results with the initial tracking errors of $e_1^T(0) = [2, -2]$ and $e_2[0] = [3, -3]$ are shown in Figure 3; $\alpha_i = 50$ and $Q_i = diag(4, 4)$ are chosen to solve the algebraic Ricatti equation.

For the proposed continuous controller we consider the more general case given by case (2) in Section 4, that is, only a bound on the interconnections, w_i, is known. The initial tracking errors for each subsystem are within the region of attraction given by $R_{ti} = 5$, the right-hand-side of (30) is $\gamma_i + N2^{p_{mi}}\sigma_{2i}\sum_{k=1}^{p_{mi}} R^{k-1} = 50$ with $\gamma_i = 2$. The choice of $k_i = [40 \quad 40]^T,\ \ i = 1, 2$ and Q_i equal to the identity matrix results in the ratio $\sigma_{qi}/\sigma_{1i} = 91.74, i = 1, 2$, which satisfies the sufficient condition. Thus, the control input for each subsystem is:

$$u_i(t) = r_i(t) - v_i(z_m(t)) - k_i^T z_i(t) \tag{41}$$

where $v_i(z_m)$ is chosen as given by (28) for the continuous control with $\epsilon_i = 0.001$. The simulation results for the proposed continuous controller are shown in Figure 4.

6 Conclusions

In this paper we have considered semi-globally stable decentralized control of a class of large-scale interconnected systems. For the regulation case, we have shown that a linear decentralized controller is able to attain semi-global exponential convergence of the initial errors to zero. For the tracking case, we have shown that a linear decentralized controller together with a discontinuous bounded term based on desired state trajectories is capable of assuring exponential convergence of tracking errors to zero; we have also designed a continuous controller which assures that every subsystem tracking error is ultimately bounded within a certain neighborhood of zero. The advantage of the proposed semi-global controllers over the global high-order controllers in the existing literature is that the magnitude of the control input for the semi-global controllers is much smaller than the global controller. Future research will focus on extensions of the ideas in this paper to large-scale systems that do not satisfy matching conditions; and also application of the decentralized controllers to practical engineering systems such as web handling systems.

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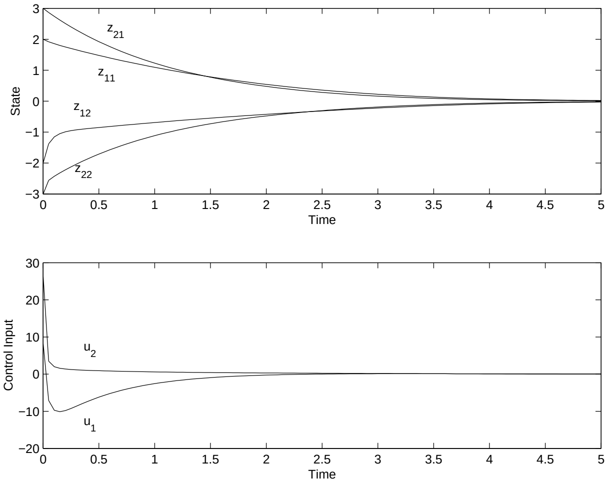

Figure 1: Regulation. Response using the global controller.

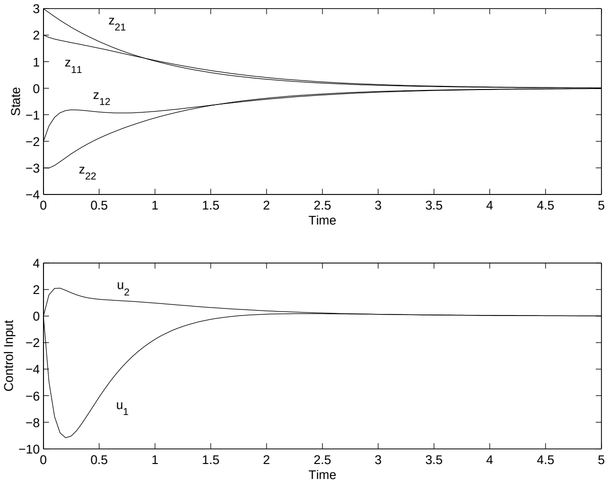

Figure 2: Regulation. Response using the linear decentralized controller.

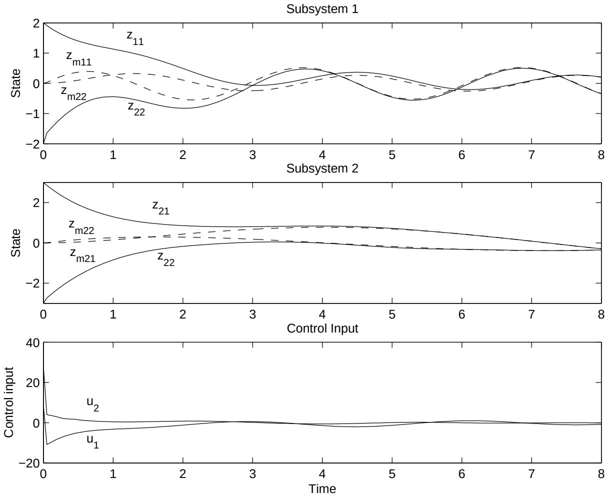

Figure 3: Tracking. Response using the higher-order global controller.

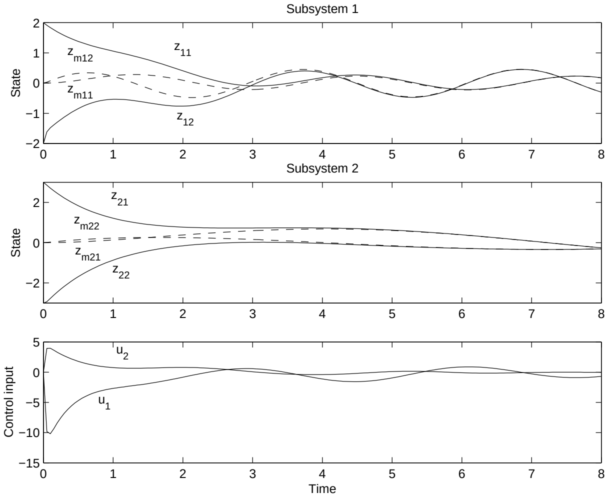

Figure 4: Tracking. Response using the proposed continuous controller.