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Distributed Formation Control of Multiple Aircraft Using Constraint Forces

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***Abstract*— A new approach for formation flight control of multiple aircraft is presented. Constraint forces are used to derive the dynamics of a constrained, multi-body system. A stable, distributed control algorithm is designed based on the information flow graph for a group of aircraft. The aircraft will achieve a particular formation while ensuring accurate navigation of the entire group. It is assumed that uncertainty exists in the drag coefficient of each aircraft. An adaptation algorithm is developed to compensate for the uncertainty and estimate the drag coefficient. The advantage of the proposed distributed control algorithm is that it allows the addition/removal of other aircraft into/from the formation seamlessly with simple modifications of the control input. Furthermore, the algorithm provides inherent scalability. Simulations were conducted to verify the proposed approach.**

I. Introduction

The problem of autonomous formation flight is an important research area in the aerospace field. Multiple aircraft flight in formations with defined geometries leads to many advantages and applications. For example, energy saving from vortex forces and fuel efficiency via induced drag reduction. Formation flying can also be used for airborne refueling and quick deployment of troops and vehicles. Moreover, many aircraft involved in a mission can be better managed if they fly in a specific formation rather than in an undefined structure.

The main goal of the formation flight is to achieve a desired group formation shape, while controlling the overall behavior of the group maneuver of multiple aircraft. Most of the formation flight strategies consist of variations in the leader/wingman formation [1]. To overcome the limitation of the leader/wingman strategy, Giulietti et al in [2] proposed a strategy in which each aircraft is not required to keep its position with respect to the formation leader, but an imaginary point in the formation. In addition to research on aircraft formation flight, there have also been a number of studies on coordinating multiple mobile robots and spacecrafts. Numerous control schemes have been proposed for the multi-agent coordination problem.

A popular approach to achieve coordination is to consider the mechanical nature of the systems and shape the dynamics of the formation using potential fields. Some recent work using potential functions can be found in [3], [4]. The basic idea is to create an energy like function in terms of the distance constraints between vehicles; the negative gradient of the potential function is used as a restoring force on each vehicle to achieve coordination. In [3], an approach for distributed control of multiple agents by using artificial potential functions and virtual leaders was given. The individual agent behaves according to the interaction forces generated by sensing the positions of neighboring agents. In [4], a specific potential function which is a function of the distance constraints of the desired formation is used. The idea of artificial potential functions for obstacle avoidance for multiple vehicles with kinematic models can be found in [5]. Instead of relying on repelling potential forces, [6] presented a control law for multiple systems based on gyroscopic forces for collision and obstacle avoidance; the gyroscopic forces were used for obstacle avoidance without affecting the global potential function.

The new approach for formation flight of multiple aircraft presented in this paper uses the theory of constraint forces to build a formation from arbitrary initial conditions for the aircraft. The idea of constrained dynamics for a system of multiple bodies with constraints is that the description of the system not only includes the external forces acting on the bodies but also the constraint forces which limit the motion of the system to be consistent with the constraints. The constraints on the system are imposed by adding a set of forces to the governing equations which keep formation separation constraints satisfied for all time [7]. The key idea of the proposed work is to use the theory of constraint forces to determine the total force required on each aircraft to maintain the formation separation. The force required to maintain the constraints for formation of a group of aircraft is calculated directly. A centralized control strategy with full information for formation of a group of vehicles using the notion of constraint forces was given in [8].

In the potential (or penalty) function approach, the square of the constraint function (or some other appropriate positive function of constraints) is treated as a potential energy. A formation keeping force that is proportional to the gradient of the potential energy is used. Since these restoring forces are regular forces which rely on displacements, they compete with every other applied force. The advantage of the constraint force approach is that the calculated constraint forces cancel only those applied forces that act against the constraints. The main contribution of this paper is in the development of a stable, scalable, and distributed control

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algorithm for multiple aircraft using constraint forces that will simultaneously achieve, and maintain, a given formation together with tracking of a desired group trajectory. Moreover, the control algorithm is adaptive in the sense that the uncertain drag coefficient of each aircraft is estimated by an adaptive algorithm.

The rest of the paper is organized as follows. Section II gives the point-mass model for each aircraft. In Section III, the distributed formation flight control is developed. Section IV gives simulation results on an example of three aircraft. Conclusions are given in Section V.

II. Aircraft Models

We consider a group of n aircraft. In this work, the following point-mass aircraft model is considered [9]:

$$\dot{x}_i = V_i \cos\chi_i \cos\gamma_i \quad (1)$$

$$\dot{y}_i = V_i \sin\chi_i \cos\gamma_i \quad (2)$$

$$\dot{h}_i = V_i \sin\gamma_i \quad (3)$$

$$\dot{V}_i = -g\sin\gamma_i + \frac{1}{m_i}(T_i - D_i) \quad (4)$$

$$\dot{\chi}_i = \frac{L_i \sin\mu_i}{m_i V_i \cos\gamma_i} \quad (5)$$

$$\dot{\gamma}_i = \frac{1}{m_i V_i}(L_i\cos\mu_i - m_i g\cos\gamma_i) \quad (6)$$

where $i = 1, 2, \ldots, n$. The coordinates x_i and y_i and the altitude h_i specify the position of the center of gravity of the i-th aircraft in an earth-based reference frame. The orientation of the aircraft, i.e, the direction of the velocity vector, is denoted by the heading angle χ_i, flight path angle γ_i, and bank angle μ_i. Heading angle is the angle between the projection of the velocity vector onto the xy plane and the x-axis. The angle between the velocity vector and its projection onto the xy plane is the flight path angle. The bank angle is then the rotation around the velocity vector. The aircraft velocity V_i is assumed to be equal to the airspeed. In Fig. 1, T_i is the engine thrust, D_i is the drag, L_i is the lift, m_i is the aircraft mass, and g is the acceleration due to gravity. The thrust depends on the altitude h_i, velocity V_i, and the throttle setting η_i by a known relationship $T_i = T_i(h_i, V_i, \eta_i)$. Also, it is assumed that the drag is a function of h_i, V_i and L_i, that is, $D_i = D_i(h_i, V_i, L_i)$.

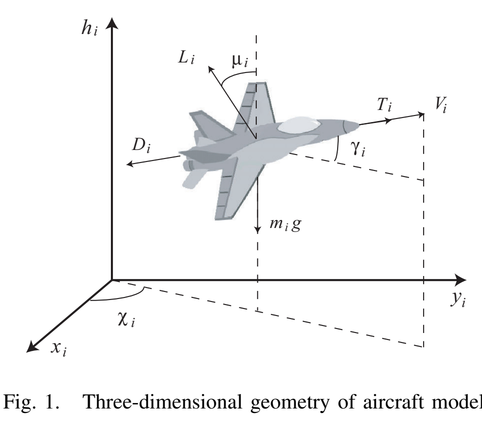

Fig. 1. Three-dimensional geometry of aircraft model.

In the model, the engine thrust T_i, the lift L_i, and the bank angle μ_i are the control variables for the aircraft. The drag can be expressed as a function of a non-dimensional drag coefficient C_{D_i} in the form of $D_i = \frac{1}{2}\rho V_i^2 S_i C_{D_i}$, where S_i is an aerodynamic reference area of the aircraft, and the quantity ρ is the average density of air. For simplicity, the air density is assumed to be a constant. The drag coefficient is assumed to have a nominal component and a component which increases quadratically with the lift as $C_{D_i} = C_{D_{0i}} + K_i C_{L_i}^2$, where $C_{D_{0i}}$ is the profile drag coefficient, which is assumed to be a constant, C_{L_i} is the lift coefficient and $K_i C_{L_i}^2$ is the induced drag. Typical values for $C_{D_{0i}}$ for the whole aircraft, are of the order of 0.003 to 0.02. Replacing the lift coefficient with the load factor, the drag D_i can be computed as $D_i = \frac{1}{2}\rho V_i^2 S_i C_{D_{0i}} + 2K_i \frac{L_i^2}{\rho V_i^2 S_i}$.

It is assumed that aggressive maneuvering will not be necessary to hold a desired formation and the aircraft will operate close to wing level, steady-state flight. Therefore, any uncertainties in drag forces will dominate and be most influential to the aircraft dynamics. To compensate for uncertainties in drag, the coefficient $C_{D_{0i}}$ is considered to be an unknown parameter and be estimated with an adaptive law.

Differentiating the expressions (1)-(3) with respect to time, and substituting dynamics of V_i, χ_i and γ_i from the expressions (4)-(6), the dynamics of the position of the aircraft is given by

$$\ddot{q}_i = U_i + \Delta_i \quad (7)$$

where $q_i = [x_i, y_i, h_i]^T \in \mathbb{R}^3$ is the position and $U_i = [U_{x_i}, U_{y_i}, U_{h_i}]^T \in \mathbb{R}^3$ is the virtual control input. The uncertain quantity Δ_i in (7) can be expressed as $\Delta_i = \Psi_i C_{D_{0i}}$ where $\Psi_i \in \mathbb{R}^3$ is a known function given by

$$\Psi_i = \begin{bmatrix} -\rho V_i^2 S_i \cos\chi_i \cos\gamma_i / 2m_i \\ -\rho V_i^2 S_i \sin\chi_i \cos\gamma_i / 2m_i \\ -\rho V_i^2 S_i \sin\gamma_i / 2m_i \end{bmatrix}. \quad (8)$$

If the virtual control variables are known, the actual control variables can be obtained using the following expressions [9]:

$$\mu_i = \text{atan}\left(\frac{U_{y_i}\cos\chi_i - U_{x_i}\sin\chi_i}{\cos\gamma_i(U_{h_i} + g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}\right) \quad (9)$$

$$L_i = m_i \frac{\cos\gamma_i(U_{h_i} + g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}{\cos\mu_i} \quad (10)$$

$$T_i = m_i\left[\sin\gamma_i(U_{h_i} + g) + \cos\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)\right] + 2K_i \frac{L_i^2}{\rho V_i^2 S_i}. \quad (11)$$

Based on the pre-linearized aircraft dynamics given by (7) for each of the n aircraft, the goal is to design a distributed formation controller that achieves and maintains a given formation together with tracking of a desired group trajectory under a given information flow between different aircraft within the group.

III. Formation Control Design

The formation structural topology of the aircraft can be defined as a formation graph, which can be used to study the relative position of aircraft in the group by applying graph theory. The formation graph of n aircraft is defined as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite set of vertices (nodes) in correspondence with the n aircraft in the group and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges (i, j) representing inter-aircraft position specifications. For simplicity, the information flow graph and the formation graph are assumed to be identical.

In the constraint force approach, geometric constraints are imposed on the system of bodies (aircraft in this case) by adding a set of constraint forces to the governing equations which keep the constraints satisfied. The overall control input U_i for the ith aircraft that is required to achieve/maintain the formation and track the desired group trajectory can be expressed as

$$U_i = F_i + F_{c_i} \tag{12}$$

where F_i is the applied force per unit mass and F_{c_i} is the constraint force per unit mass that limits the motion of the system to be consistent with the constraints. To compensate for the uncertainties in the model, the applied force F_i is

$$F_i = F_{n_i} - F_{ad_i}. \tag{13}$$

where F_{n_i} is the navigational feedback control given as

$$F_{n_i} = \ddot{q}_{d_i} - c_1 e_i - c_2 \dot{e}_i \tag{14}$$

where c_1 and c_2 are positive constants, $e_i = q_i - q_{d_i}$ and $\dot{e}_i = \dot{q}_i - \dot{q}_{d_i}$ are navigational tracking errors, and $q_{d_i}, \dot{q}_{d_i}$ and $\ddot{q}_{d_i}$ are the desired position, velocity, and acceleration, respectively. The adaptive term F_{ad_i} is used to compensate for the uncertainties,

$$F_{ad_i} = \Psi_i \widehat{C}_{D_{0i}} \tag{15}$$

where $\widehat{C}_{D_{0i}}$ is the estimate of $C_{D_{0i}}$.

First, the constraint force between a pair of communicating aircraft will be designed. The application of such a force on each aircraft, in addition to the applied force (both navigation and adaptive), will ensure that the constraint is satisfied between the two aircraft. Second, the distributed control algorithm for the entire group will be developed based on the constraint force between a pair of communicating aircraft.

A. Constraint Force Between a Pair of Aircraft

Consider a pair of aircraft i and j which share an edge in the information flow graph, i.e., $(i, j) \in \mathcal{E}$. Denote the constraint corresponding to the (i, j) edge in the formation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ by $\phi_{ij}(q_i, q_j) = 0$ for any $(i, j) \in \mathcal{E}$ and $i \neq j$, with $\phi_{ij}(q_i, q_j)$ in some specific form. Define the composite position vector of two communicating aircraft by $q_{ij} = [q_i^T, q_j^T]^T \in \mathcal{R}^6$. The constraint function is defined as a function of distance between the two aircraft $\|q_i - q_j\|$,

$$\phi_{ij}(q_{ij}) = h(\|q_i - q_j\|, d_{ij}), \quad (i, j) \in \mathcal{E} \tag{16}$$

where d_{ij} is the length of the edge (i, j) which is the desired distance between the two aircraft in the formation. The structural constraint can be expressed as $\phi_{ij}(q_{ij}) = 0$. Differentiating (16) once, we get the constraint velocity as

$$\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}) = \frac{\partial \phi_{ij}(q_{ij})}{\partial q_{ij}} \dot{q}_{ij} := A_{ij}(q_{ij})\dot{q}_{ij} \tag{17}$$

where $A_{ij}(q_{ij}) = \frac{\partial \phi_{ij}(q_{ij})}{\partial q_{ij}}$ is a specially structured 1×6 matrix called the constraint matrix, which is in the form of

$$A_{ij}(q_{ij}) = \left[\begin{array}{cc} a_{ij}^T & a_{ji}^T \end{array}\right] \quad \text{with} \quad a_{ij} = -a_{ji} \in \mathbb{R}^3. \tag{18}$$

For example, if the constraint function can be defined as the function of the Euclidean distance between two aircraft, $\phi_{ij}(q_{ij}) = \|q_i - q_j\| - d_{ij}, \quad (i, j) \in \mathcal{E}$, then the constraint matrix is $A_{ij}(q_{ij}) = \left[\begin{array}{cc} \frac{(q_i - q_j)^T}{\|q_i - q_j\|} & -\frac{(q_i - q_j)^T}{\|q_i - q_j\|} \end{array}\right]$.

Differentiating $\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij})$ again, we get the constraint acceleration as

$$\ddot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}, \ddot{q}_{ij}) = \dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} + A_{ij}(q_{ij})\ddot{q}_{ij} \tag{19}$$

where $\dot{A}_{ij}(q_{ij}, \dot{q}_{ij}) = \frac{\partial \dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij})}{\partial q_{ij}}$.

Assuming that the configuration q_{ij} and the velocity $\dot{q}_{ij}$ both have the desired initial values, i.e., $\phi_{ij}(q_{ij}^0) = \dot{\phi}_{ij}(q_{ij}^0, \dot{q}_{ij}^0) = 0$, The idea behind the constraint force approach is that we need to choose $F_{c_{ij}}$ such that the constraint acceleration is identically zero, that is, $\ddot{\phi}_{ij} = 0$, then all subsequent motion is such that $\phi_{ij} = 0$.

Based on the discussion of the constrained dynamics, the dynamics of the constrained system can be written as

$$\ddot{q}_{ij} = F_{n_{ij}} + F_{c_{ij}} + \left(\Delta_{ij} - F_{ad_{ij}}\right) \tag{20}$$

where $F_{n_{ij}} = [F_{n_i}^T, F_{n_j}^T]^T$, $F_{c_{ij}} = [F_{c_i}^T, F_{c_j}^T]^T$, $F_{ad_{ij}} = [F_{ad_i}^T, F_{ad_j}^T]^T$ and $\Delta_{ij} = [\Delta_i^T, \Delta_j^T]^T$.

Substituting the constrained system dynamics (20) into (19), we get

$$\ddot{\phi}_{ij} = \dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}} + A_{ij}\left(\Delta_{ij} - F_{ad_{ij}}\right). \tag{21}$$

Ignoring the uncertainties in the constraint dynamics, we use the following nominal constraint dynamics to derive the constraint forces

$$\ddot{\phi}_{ij}^{nom} = \dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}}. \tag{22}$$

Setting (22) to be zero, the constraint force satisfies the following equation:

$$A_{ij}(q_{ij})F_{c_{ij}} = -\dot{A}_{ij}(q_{ij}, \dot{q}_{ij})\dot{q}_{ij} - A_{ij}(q_{ij})F_{ij}. \tag{23}$$

Equation (23) alone does not uniquely determine the constraint force, since we have only one equation and six unknowns (the six components of $F_{c_{ij}}$). The widely used procedure in dynamics is to use the principle of virtual work [10] ; which states that the constraint forces do not add or remove energy. Therefore, to ensure that the constraint force does no work, we require that $F_{c_{ij}}^T \dot{q}_{ij}$ be zero for every $\dot{q}_{ij}$ satisfying $\dot{\phi}_{ij}(q_{ij}, \dot{q}_{ij}) = 0$, that is,

$$F_{c_{ij}}^T \dot{q}_{ij} = 0, \ \forall \ \dot{q}_{ij} \in \{\dot{q}_{ij} \mid A_{ij}(q_{ij})\dot{q}_{ij} = 0\}. \tag{24}$$

From Eq. (24), it is clear that $F_{c_{ij}}$ must be orthogonal to the velocity vector $\dot{q}_{ij}$. Since $\dot{q}_{ij}$ must lie in the null space of $A_{ij}(q_{ij})$, the constraint force $F_{c_{ij}}$ must lie in the null space complement of $A_{ij}(q_{ij})$. Thus, the vector $F_{c_{ij}}$ satisfying Eq. (24) can be expressed in the form

$$F_{c_{ij}} = A_{ij}^T(q_{ij})\lambda_{ij} \tag{25}$$

where λ_{ij} is the Lagrange multiplier, which is obtained by substituting (25) into (23),

$$A_{ij}(q_{ij})A_{ij}^T(q_{ij})\lambda_{ij} = -\dot{A}_{ij}(q_{ij},\dot{q}_{ij})\dot{q}_{ij} - A_{ij}(q_{ij})F_{ij}. \tag{26}$$

The constraint force for aircraft i and j are then given by $F_{c_i} = [I_3,\ \emptyset_3]F_{c_{ij}}$ and $F_{c_j} = [\emptyset_3,\ I_3]F_{c_{ij}}$ where I_3 and $\emptyset_3$ are the identity and zero matrices, respectively, of dimension of three.

The entire discussion on the development of the constraint forces so far was based on the assumption that at the start of the motion of the aircraft, the constraint equations are satisfied. To consider arbitrary initial conditions for the aircraft, which do not satisfy the constraint equations, we will use the notion of feedback in the constraint acceleration equation; this will account for the mismatch in the initial condition and appropriately compute the constraint force. This idea was used to prevent numerical drift in the simulation of dynamic equations with constraints in [11]. Instead of solving for $\ddot{\phi}_{ij}^{nom} = 0$ to determine the constraint force, as it was done in our earlier work [8], the following equation will be used:

$$\ddot{\phi}_{ij}^{nom} = -k_d\dot{\phi}_{ij} - k\lambda_{ij} \tag{27}$$

where k_d and k are positive constants. Therefore, the constraint force vector for the two aircraft is calculated based on the following equations:

$$F_{c_{ij}} = A_{ij}^T\lambda_{ij}, \tag{28}$$

$$\lambda_{ij} = \frac{1}{k + A_{ij}A_{ij}^T}\left(-\dot{A}_{ij}\dot{q}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij}\right). \tag{29}$$

The constraint force for aircraft i and j are then given by $F_{c_i} = [I_3,\ \emptyset_3]F_{c_{ij}}$ and $F_{c_j} = [\emptyset_3,\ I_3]F_{c_{ij}}$. Note that the constraint forces on the two aircraft satisfy $F_{c_i} = -F_{c_j}$. Since they are internal forces – addition of the i-th and j-th dynamics will result in cancelation of these forces for the two aircraft case.

We consider the following Lyapunov function candidate

$$E_{ij} = \frac{1}{2}kc_1e_{ij}^Te_{ij} + \frac{1}{2}k\dot{e}_{ij}^T\dot{e}_{ij} + \frac{1}{2}\dot{\phi}_{ij}^2 + \frac{1}{2}\Gamma\widetilde{C}_{D_{0ij}}^T\widetilde{C}_{D_{0ij}} \tag{30}$$

where Γ is a positive constant, $e_{ij} = [e_i^T, e_j^T]^T$, and $\widetilde{C}_{D_{0ij}} = [\widetilde{C}_{D_{0i}}, \widetilde{C}_{D_{0j}}]^T$ with $\widetilde{C}_{D_{0i}} = \widehat{C}_{D_{0i}} - C_{D_{0i}}$. The derivative of E_{ij} with respect to time is given by

$$\begin{aligned}\dot{E}_{ij} &= kc_1\dot{e}_{ij}^Te_{ij} + k\dot{e}_{ij}^T\left(F_{n_{ij}} + F_{c_{ij}} + (\Delta_{ij} - F_{ad_{ij}}) - \ddot{q}_{d_{ij}}\right)\\ &\quad + \dot{\phi}_{ij}\left(\dot{A}_{ij}\dot{q}_{ij} + A_{ij}F_{n_{ij}} + A_{ij}F_{c_{ij}} + A_{ij}(\Delta_{ij} - F_{ad_{ij}})\right)\\ &\quad + \Gamma\widetilde{C}_{D_{0ij}}^T\dot{\widetilde{C}}_{D_{0ij}}\\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})(\Delta_{ij} - F_{ad_{ij}})\\ &\quad + \Gamma\widetilde{C}_{D_{0ij}}^T\dot{\widetilde{C}}_{D_{0ij}}.\end{aligned}$$

Note that $A_{ij} = [a_{ij}^T, a_{ji}^T]$ with $a_{ij} = -a_{ji}$, then

$$\begin{aligned}\dot{E}_{ij} &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 + (k\dot{e}_{ij}^T + \dot{\phi}_{ij}A_{ij})\begin{bmatrix} -\Psi_i\widetilde{C}_{D_{0i}} \\ -\Psi_j\widetilde{C}_{D_{0j}}\end{bmatrix}\\ &\quad + \Gamma\widetilde{C}_{D_{0ij}}^T\dot{\widetilde{C}}_{D_{0ij}}\\ &= -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2\\ &\quad + \begin{bmatrix} \widetilde{C}_{D_{0i}}^T & \widetilde{C}_{D_{0j}}^T\end{bmatrix}\begin{bmatrix} -k\Psi_i^T\dot{e}_i - \dot{\phi}_{ij}\Psi_i^Ta_{ij} + \Gamma\dot{\widetilde{C}}_{D_{0i}} \\ -k\Psi_j^T\dot{e}_j - \dot{\phi}_{ij}\Psi_j^Ta_{ji} + \Gamma\dot{\widetilde{C}}_{D_{0j}}\end{bmatrix}.\end{aligned} \tag{31}$$

To estimate the unknown parameters $C_{D_{0i}}$ and $C_{D_{0j}}$, we use the gradient projection algorithm [12]. Consider a convex parameter set Π_i given by

$$\widehat{C}_{D_{0i}} \in \Pi_i \Longleftrightarrow |\widehat{C}_{D_{0i}} - \rho_i| < \sigma_i \tag{32}$$

with ρ_i some given real number. Consider the function

$$\mathcal{P}_i(\widehat{C}_{D_{0i}}) = \frac{2}{\epsilon_i}\left[\left|\frac{\widehat{C}_{D_{0i}} - \rho_i}{\sigma_i}\right|^q - 1 + \epsilon_i\right] \tag{33}$$

where $0 < \epsilon_i < 1$ and $q \geq 2$. Consider the 'smooth projection' Proj $(\cdot)$, which will be used to estimate $\widehat{C}_{D_{0i}}$ while maintaining it in Π_i:

$$\text{Proj}(\widehat{C}_{D_{0i}}, y_i) = \begin{cases} y_i, & \text{if } \mathcal{P}_i < 0\\ y_i, & \text{if } \mathcal{P}_i = 0 \text{ and } \nabla_{\mathcal{P}_i}^T y_i \leq 0\\ y_i - \frac{\mathcal{P}_i\nabla_{\mathcal{P}_i}\nabla_{\mathcal{P}_i}^T}{\|\nabla_{\mathcal{P}_i}\|^2}y_i, & \text{otherwise}\end{cases} \tag{34}$$

where $\nabla_{\mathcal{P}_i} = \left[\partial\mathcal{P}_i(\widehat{C}_{D_{0i}})/\partial\widehat{C}_{D_{0i}}\right]^T$ is a column vector. Based on the smooth projection defined above, $\widehat{C}_{D_{0i}}$ and $\widehat{C}_{D_{0j}}$ are estimated by

$$\dot{\widehat{C}}_{D_{0i}} = \Gamma^{-1}\text{Proj}\left(\widehat{C}_{D_{0i}}, k\Psi_i^T\dot{e}_i + \dot{\phi}_{ij}\Psi_i^T\left[I_3,\ \emptyset_3\right]A_{ij}^T\right)$$

$$\dot{\widehat{C}}_{D_{0j}} = \Gamma^{-1}\text{Proj}\left(\widehat{C}_{D_{0j}}, k\Psi_i^T\dot{e}_i + \dot{\phi}_{ji}\Psi_i^T\left[I_3,\ \emptyset_3\right]A_{ji}^T\right)$$

where the corresponding projection for the j-th aircraft is given by replacing the index i by j in (32), (33) and (34). Substitution of the adaptation law in (31) gives

$$\dot{E}_{ij} = -kc_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 \leq 0. \tag{35}$$

Therefore, E_{ij} is a Lyapunov function. As a result, e_{ij}, $\dot{e}_{ij}$, $\dot{\phi}_{ij}$, and $\widetilde{C}_{D_{0ij}}$ are bounded. From (30) and (35), we can conclude that $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ are square integrable signals. Further, from the dynamics of the constraint and the tracking error,

$$\ddot{\phi}_{ij} = -k_d\dot{\phi}_{ij} - k\lambda_{ij} - A_{ij}\begin{bmatrix}\Psi_i\widetilde{C}_{D_{0i}}\\ \Psi_j\widetilde{C}_{D_{0j}}\end{bmatrix}, \tag{36}$$

$$\ddot{e}_{ij} = -c_1e_{ij} - c_2\dot{e}_{ij} + A_{ij}^T\lambda_{ij} - \begin{bmatrix}\Psi_i\widetilde{C}_{D_{0i}}\\ \Psi_j\widetilde{C}_{D_{0j}}\end{bmatrix}, \tag{37}$$

we can conclude that both $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ are bounded. Therefore, $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ converge to zero asymptotically by invoking Barbalat's lemma. Further, we can show via direct calculation that $\dddot{\phi}_{ij}$ and $\dddot{e}_{ij}$ are bounded by differentiating Eqs. (36) and (37). Therefore, the signals $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ asymptotically converge to zero. From Eqs. (36) and (37), we can conclude that e_{ij}, $\widetilde{C}_{D_{0i}}$ and $\widetilde{C}_{D_{0j}}$ converge to the relationship given by

$$e_{ij} = -\frac{1}{c_1}(I_6 + A_{ij}^T A_{ij}/k) \begin{bmatrix} \Psi_i \widetilde{C}_{D_{0i}} \\ \Psi_j \widetilde{C}_{D_{0j}} \end{bmatrix}. \quad (38)$$

Therefore, as expected, the position tracking error depends on the uncertain parameter estimation error. The error can be decreased by increasing c_1. Moreover, from the definition of the constraint vector, $\phi_{ij}(q_{ij}) = \| q_i - q_j \| - d_{ij} = \| (e_i - e_j) + (q_{d_i} - q_{d_j}) \| - d_{ij} \leq \|e_i - e_j\|$ since $\|q_{d_i} - q_{d_j}\| = d_{ij}$.

B. Distributed Control Algorithm for Multiple Aircraft

As in the previous section, we consider the applied force F_i acting on the i-th aircraft to be given by (13), with the navigational feedback control F_{n_i} given by (14), and the adaptive control F_{ad_i} given by (15). The adaptation law is given as

$$\dot{\widehat{C}}_{D_{0i}} = \Gamma^{-1}\text{Proj}\left(\widehat{C}_{D_{0i}}, k\Psi_i^T \dot{e}_i + \sum_{(i,j)\in\mathcal{E}} \dot{\phi}_{ij} \Psi_i^T \left[I_3,\ \emptyset_3\right] A_{ij}^T\right). \quad (39)$$

The constraint force acting on the i-th aircraft is chosen as the total of the constraint force contribution from all aircraft which directly communicate with it, i.e., F_{c_i} is given by

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} \frac{\left[I_3,\ \emptyset_3\right] A_{ij}^T}{k + A_{ij} A_{ij}^T} \left(-\dot{A}_{ij}\dot{q}_{ij} - A_{ij} F_{n_{ij}} - k_d \dot{\phi}_{ij}\right). \quad (40)$$

To show that this control input tracks the desired navigation trajectories and achieves, and maintains, the given formation, we consider the following composite Lyapunov function candidate:

$$E = \frac{1}{2}\sum_{i=1}^{n}\left(kc_1 e_i^T e_i + k\dot{e}_i^T \dot{e}_i + \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}^2 + \Gamma \widetilde{C}_{D_{0i}}^2\right). \quad (41)$$

Taking the derivative of E with respect to time, substituting the control law (12) and the dynamics of the aircraft (7) into $\dot{E}$, and simplifying, we get

$$\begin{aligned} \dot{E} &= -kc_2 \sum_{i=1}^{n} \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}^2 \\ &+ k\left(\sum_{i=1}^{n} \dot{q}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}\lambda_{ij}\right) - k\sum_{i=1}^{n} \dot{q}_{d_i}^T F_{c_i} \\ &- \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij} \left[\ a_{ij}^T \quad a_{ji}^T\ \right] \begin{bmatrix} \Psi_i \widetilde{C}_{D_{0i}} \\ \Psi_j \widetilde{C}_{D_{0j}} \end{bmatrix} \\ &- k\sum_{i=1}^{n} \dot{e}_i^T \Psi_i \widetilde{C}_{D_{0i}} + \sum_{i=1}^{n} \Gamma \widetilde{C}_{D_{0i}} \dot{\widetilde{C}}_{D_{0i}}. \end{aligned} \quad (42)$$

In Eq. (42) , we can show that the third and fourth terms are identically equal to zero. Note that

$$\begin{aligned} &\sum_{i=1}^{n} \dot{q}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}\lambda_{ij} \\ &= \sum_{i=1}^{n} \dot{q}_i^T \sum_{(i,j)\in\mathcal{E}} [I_3,\ \emptyset_3] A_{ij}^T \lambda_{ij} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} A_{ij}\dot{q}_{ij}\lambda_{ij} \\ &= \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}} \dot{q}_i^T a_{ij}\lambda_{ij} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} (a_{ij}^T \dot{q}_i + a_{ji}^T \dot{q}_j)\lambda_{ij}. \end{aligned}$$

Since $\lambda_{ij} = \lambda_{ji}$, we have $\sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} (a_{ij}^T \dot{q}_i + a_{ji}^T \dot{q}_j)\lambda_{ij} = \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}} a_{ij}^T \dot{q}_i \lambda_{ij}$. Thus, we have

$$\sum_{i=1}^{n} \dot{q}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}\lambda_{ij} = 0. \quad (43)$$

Further, since $\dot{q}_{d_i}$ is the desired velocity, it must satisfy $\dot{q}_{d_{ij}}^T A_{ij}^T = 0$ for any $i \neq j$. Hence, $\sum_{i=1}^{n} \dot{q}_{d_i}^T F_{c_i} = \sum_{i=1}^{n} \sum_{\substack{(i,j)\in\mathcal{E} \\ j>i}} \dot{q}_{d_{ij}}^T A_{ij}^T \lambda_{ij} = 0$. Therefore,

$$\begin{aligned} \dot{E} &= -kc_2 \sum_{i=1}^{n} \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}^2 + \sum_{i=1}^{n} \Gamma \widetilde{C}_{D_{0i}} \dot{\widetilde{C}}_{D_{0i}} \\ &- \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}} \dot{\phi}_{ij} \widetilde{C}_{D_{0i}}^T \Psi_i^T a_{ij} - k\sum_{i=1}^{n} \widetilde{C}_{D_{0i}}^T \Psi_i^T \dot{e}_i. \end{aligned} \quad (44)$$

Using the adaptive law (39) we have

$$\dot{E} = -kc_2 \sum_{i=1}^{n} \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E}, j>i} \dot{\phi}_{ij}^2 \leq 0. \quad (45)$$

Using the same arguments as in the previous section, we can conclude that all signals are bounded, the signals $\dot{e}_i$ and $\dot{\phi}_{ij}$ asymptotically converge to zero, and e_i and ϕ_{ij} are bounded by a function of the parameter estimation error, similar to Eq. (38). The results of this section are summarized in the following theorem.

Theorem 1: For a group of aircraft given by the dynamics (1)-(6), the choice of the following control algorithm:

$$\mu_i = \text{atan}\left(\frac{U_{y_i}\cos\chi_i - U_{x_i}\sin\chi_i}{\cos\gamma_i(U_{h_i} + g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}\right)$$

$$L_i = m_i \frac{\cos\gamma_i(U_{h_i} + g) - \sin\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)}{\cos\mu_i}$$

$$\begin{aligned} T_i &= m_i\left[\sin\gamma_i(U_{h_i} + g) + \cos\gamma_i(U_{x_i}\cos\chi_i + U_{y_i}\sin\chi_i)\right] \\ &\quad + 2K_i \frac{L_i^2}{\rho V_i^2 S_i} \end{aligned}$$

$$U_i = F_{n_i} - F_{ad_i} + F_{c_i}, \text{ with } U_i = [U_{x_i}, U_{y_i}, U_{h_i}]^T$$

$$F_{n_i} = \ddot{q}_{d_i} - c_1 e_i - c_2 \dot{e}_i$$

$$F_{ad_i} = \Psi_i \widehat{C}_{D_{0i}}$$

$$\dot{\widehat{C}}_{D_{0i}} = \Gamma^{-1}\text{Proj}\left(\widehat{C}_{D_{0i}}, k\Psi_i^T \dot{e}_i + \sum_{(i,j)\in\mathcal{E}} \dot{\phi}_{ij}\Psi_i^T \left[I_3,\ \emptyset_3\right] A_{ij}^T\right)$$

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} \frac{[I_3,\ \emptyset_3]\, A_{ij}^T}{k + A_{ij}A_{ij}^T}\left(-\dot{A}_{ij}\dot{q}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij}\right)$$

will ensure that all signals are bounded, the signals $\dot{e}_i$ and $\dot{\phi}_{ij}$ asymptotically converge to zero.

IV. Simulations

The performance of three aircraft flying in V-formation along a straight path is evaluated in the three dimensional case. Each aircraft in the group starts from an arbitrary location which does not satisfy the constraint equations. The desired formation is defined as a V-shape with length of edges equal to 100 m, and at the same height of 300 m. The desired navigation trajectory for each vehicle within the V-formation is taken as a straight line with constant velocity 40 m/s. The information flow graph is defined as aircraft 3 communicating with aircraft 1 and 2, while no communication between aircraft 1 and 2. The initial positions of the three aircraft are given by $q_1(0) = [200, 0, 125]^T$ m, $q_2(0) = [-100, 125, 125]^T$ m, $q_3(0) = [100, 220, 180]^T$ m, and initial heading angles and flight path angles are zero. The drag coefficients are $C_{D_{0i}} = 0.02$ and $K_i = 0.25$ for each aircraft. To evaluate the performance of the adaptive controller, the initial value of the drag coefficient estimate $\widehat{C}_{D_{0i}}$ is taken as 0.016 which reflects as a 20% uncertainty on the true value. The adaptation gain value is chosen as $\Gamma = 10000$. The lower and upper bounds on the parameter for the projection algorithm are chosen as 0.014 and 0.026, respectively, and the tolerance is chosen as $\epsilon = 0.25$. The gain parameters for each aircraft in the distributed control algorithm are selected as $c_1 = 2$, $c_2 = 2.8$, $k_d = 2.6$, and $k = 2.8$. The simulation result is shown as a three-dimensional view in Fig. 2. Each aircraft in the group starts at the initial position denoted by $\circ$, and reaches a desired triangle formation while approaching the desired navigation trajectory. The corresponding inter-aircraft distance is shown in Figure 3.

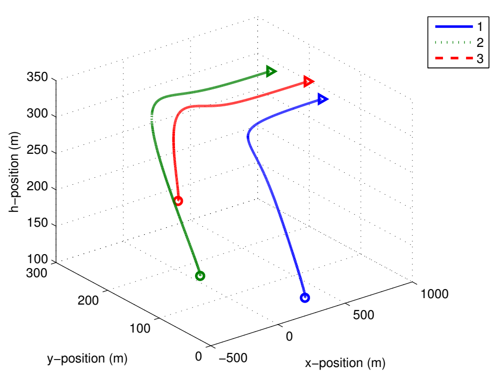

Fig. 2. Three-dimensional view of triangle formation for three aircraft.

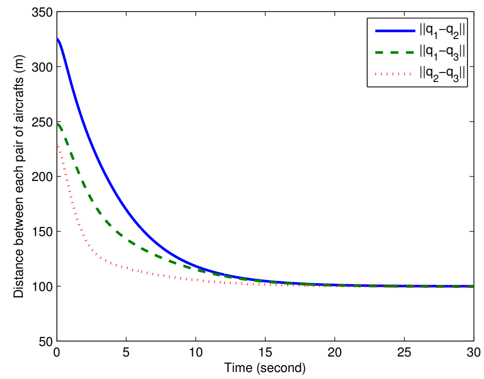

Fig. 3. Distance between pairs of aircraft.

V. Conclusions

Based on the notion of constraint forces, we have developed a stable, distributed control algorithm for multiple aircraft formation flight. The point-mass dynamics of each aircraft is feedback linearized with the output as positions of the center of gravity of the aircraft in an earth-based reference frame. Given a formation, an information flow pattern, and a desired trajectory, the distributed control algorithm developed for each aircraft in the group is capable of achieving and maintaining the formation along the desired group trajectory. Moreover, the adaptive control law is applied in the external forces to compensate for the drag coefficient parameter uncertainty. Simulation results on an example formation of a group of three aircraft were shown to corroborate the proposed algorithm.

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