Adaptive Control of Time-Varying Mechanical Systems: Analysis and Experiments

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***Abstract*—Adaptive control of time-varying mechanical systems is considered in this paper. A new adaptive controller for time-varying mechanical systems is proposed based on two assumptions. First, the dynamics of time-varying mechanical systems is derived under the assumption that the generalized constraints on the system do not depend on time but the system parameters such as masses and payloads are time-varying. Second, the time-varying parameters are given by a group of known bounded time functions and unknown constants. It is shown that the proposed adaptive controller results in a stable closed-loop system. Further, if the desired trajectory of the system is periodic, a time-scaling technique of mapping one cycle period of the desired trajectory into a unit interval is proposed to provide robustness to the parameter adaptation algorithm. An experimental platform consisting of a two-link robot with a time-varying payload is developed to test the proposed adaptive controller. The experimental platform mimics robotic pouring and filling operations in industry. Comparative experimental results demonstrate the effectiveness of the proposed design.**

***Index Terms*—Adaptive control, experiments, mechanical systems, mechatronics, time-varying systems.**

I. Introduction

RESEARCH in the area of trajectory tracking control of mechanical systems has been widespread. Many industrial tasks of robot manipulators such as material handling, transportation, part assembly, etc. involve such a problem. A large number of control designs for mechanical systems exists in literature that works well with both known and unknown constant parameters. However, in many situations, some of the unknown parameters, especially the mass of the payload or the mass of the links, may be time varying. Examples of such operations include robotic pouring and filling operations. Many adaptive control algorithms in literature are based on the assumption that the parameters are constant or slowly time varying. However, if the parameter change is significant then the robot dynamic model that is developed for the constant parameter case cannot be used to describe the dynamic behavior under time-varying parameters. Further, adaptive control algorithms developed for the constant parameter model may prove to be unstable if used for mechanical systems with time-varying parameter. The focus of this work is to develop an adaptive controller for mechanical systems with time-varying parameters and conduct experiments to validate the proposed controller.

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Adaptive control strategies for the constant parameter case can be found in [5] and its references. Limited work in the area of control of time-varying mechanical systems exists in the literature. Constrained Lagrangian dynamics of discrete systems can be found in [1] and [2]. In [3], a robust switching controller is designed for the time-varying parameter model of the robot manipulators performing path tracking tasks. Properties of the element-by-element product of matrices are used to isolate the time-varying parameters from the inertia matrix. A robust adaptive control for robot manipulators consisting of slowly time-varying parameters is presented in [4].

In this work, a dynamic model for time-varying mechanical systems is derived based on past work [1], [2] to illustrate the difference in the models between constant and time-varying cases. This model is based on the assumption that the generalized constraints are independent of time but the payload masses and/or link masses can be time varying. Compared with the dynamic model for constant parameters, the dynamic model for time-varying parameters contains extra terms that are related to the rate of change of the time-varying parameters. Some of the dynamic model properties for the time-varying parameter case are also different from those of the constant parameter case. An adaptive controller for the time-varying mechanical system is proposed under the assumption that the time-varying parameters are linearly parameterized by a group of known bounded time functions and unknown constants. Asymptotic stability of the closed-loop system under the proposed adaptive controller is shown.

To test the adaptive controller, a novel experimental platform to mimic filling/pouring operations is designed. This platform consists of a two-link robot with a time-varying payload. The time-varying nature of the payload is obtained by pumping fluid in/out of a cylindrical vessel carried by the robot manipulator, while the manipulator is in motion. Pumping of fluid in/out of the vessel leads to the payload mass being explicitly dependent on time. As a result, the regressor matrix in the adaptation law depends on time explicitly and is unbounded. To circumvent this problem, a notion of time-scaling is introduced, which involves mapping a cycle period of the desired trajectory to the unit interval [0, 1]. Successful experimental results validate the proposed adaptive design. A constant parameter model-based adaptive control algorithm is also implemented for the time-varying mechanical system to compare with the proposed adaptive controller.

The rest of the paper is organized as follows. In Section II, the dynamic model for time-varying mechanical system is derived, and properties of this model are stated. The adaptive controller is given in Section III. Stability of the closed-loop system with the proposed adaptive controller is also shown in Section III. In Section IV, an experimental platform is described. Dynamic equations for the two-link robot example with time-varying payload are given in Section IV-A. In Section IV-B, a time-scaling technique is given and the adaptive controller for the two-link robot is derived. Experimental results and discussions are summarized in Section IV-C. Section V gives the conclusions of this work.

II. Dynamic Modeling of Time-Varying Mechanical Systems

For a mechanical system with constant parameters, such as masses and lengths, the Lagrange's equations of motion are given by

$$H(q)\ddot{q} + N(q, \dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where

$q \in \mathbb{R}^n$	generalized coordinates;
$H(q) \in \mathbb{R}^{n\times n}$	positive definite inertia matrix;
$N(q, \dot{q}) \in \mathbb{R}^{n\times n}$	matrix composed of Coriolis and centrifugal terms;
$g(q) \in \mathbb{R}^n$	gravity vector;
$\tau \in \mathbb{R}^n$	vector of control inputs.

If the elements of matrix $N(q, \dot{q})$ are derived using Christoffel's symbols, then it is well known that the matrix $\dot{H} - 2N$ is skew-symmetric. This is not true if the parameters of the system are time-varying [2], [3]. The dynamics given by (1) no longer represents dynamic behavior when the inertial parameters of the mechanical system are time-varying. To illustrate the differences between models with constant inertial parameters and time-varying inertial parameters, the dynamic model for time-varying mechanical systems is derived. A complete derivation can be found in [2].

Consider a mechanical system of L particles whose position is given by the Cartesian coordinates x_k $(k = 1, \ldots, 3L)$ with respect to an inertial frame. The Cartesian coordinate of any particle can be expressed in terms of the generalized coordinates $(q_1, q_2, \ldots, q_n, t)$, as $x_k(q_1, q_2, \ldots, q_n, t)$. Then the velocity is given by

$$v_k = \frac{dx_k}{dt} = \sum_{j=1}^{n} \frac{\partial x_k}{\partial q_j}\dot{q}_j + \frac{\partial x_k}{\partial t}. \tag{2}$$

The total kinetic energy of the system is

$$T = \tfrac{1}{2}\sum_{k=1}^{3L} m_k(t) v_k^2 \tag{3}$$

$$= \frac{1}{2}\sum_{k=1}^{3L} m_k(t)\left(\sum_{j=1}^{n} \frac{\partial x_k}{\partial q_j}\dot{q}_j + \frac{\partial x_k}{\partial t}\right)^2. \tag{4}$$

Carrying out the expansion, T can be written as the sum of three homogeneous functions of the generalized velocities, $T = T_0 + T_1 + T_2$, where

$$T_0 = \frac{1}{2}\sum_{k=1}^{3L} m_k(t)\left(\frac{\partial x_k}{\partial t}\right)^2 \tag{5}$$

$$T_1 = \sum_{i=1}^{n} a_i \dot{q}_i \tag{6}$$

$$T_2 = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n} m_{ij}\dot{q}_i\dot{q}_j \tag{7}$$

where

$$m_{ij} = m_{ji} = \sum_{k=1}^{3L} m_k \frac{\partial x_k}{\partial q_i}\frac{\partial x_k}{\partial q_j} \tag{8}$$

$$a_i = \sum_{k=1}^{3L} m_k \frac{\partial x_k}{\partial q_i}\frac{\partial x_k}{\partial q_j} \tag{9}$$

where

$m_1 = m_2 = m_3$	mass of the first particle;
(x_1, x_2, x_3)	position of the first particle;
$m_4 = m_5 = m_6$	mass of the second particle;
(x_4, x_5, x_6)	position of the second particle, and so on.

Note that T_0 is independent of the generalized velocities, T_1 is linear in the generalized velocities, and T_2 is quadratic in the generalized velocities. If the generalized constraints do not contain time explicitly, i.e., $\partial x_k/\partial t = 0$, then the total kinetic energy is a homogeneous quadratic form in the generalized velocities, i.e., $T = T_2$. For serial manipulators, this would imply that the length of the links should be fixed. For the mechanical systems considered in this paper, it is assumed that the generalized constraints do not explicitly depend on time but the system masses and payloads are time-varying. The time-varying nature of the mechanical system is due to time-varying masses and payloads; see [1] and [2] for a detailed explanation of various types of constrained systems and their dynamics.

Let $\phi(t) \in \mathbb{R}^p$ be a vector of parameters of the system, both constant and time-varying. Then $m_{ij} = m_{ij}(q, \phi)$. Using Lagrange's equations, the dynamic equations are [2]

$$\sum_{j=1}^{n} m_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{l=1}^{n}[jl, i]\dot{q}_j\dot{q}_l + \sum_{j=1}^{n}\left(\sum_{k=1}^{p}\frac{\partial m_{ij}}{\partial \phi_k}\dot{\phi}_k\right)\dot{q}_j + \frac{\partial P(q, \phi)}{\partial q_i} = 0, \qquad i = 1, \ldots, n \tag{10}$$

where $P(q, \phi)$ is the potential energy of the system, and

$$[jl, i] = \frac{1}{2}\left(\frac{\partial m_{ij}}{\partial q_l} + \frac{\partial m_{il}}{\partial q_j} - \frac{\partial m_{jl}}{\partial q_i}\right). \tag{11}$$

In matrix form, the dynamic equations of the system are

$$M(q, \phi)\ddot{q} + C(q, \dot{q}, \phi)\dot{q} + F(q, \dot{\phi})\dot{q} + g(q, \phi) = \tau \tag{12}$$

where τ is a vector of external control inputs and $g(q) = \partial P(q, \phi)/\partial q$. Notice that there is an additional term, $F(q, \dot{\phi})\dot{q}$, in (12) when compared with the dynamics with constant parameters, (1).

A. Dynamic Model Properties

The properties of the dynamic model (12) are as follows.

Property I: The inertia matrix, $M(q, \phi)$, of the time-varying mechanical system is a symmetric positive definite matrix. Assuming $\phi(t)$ is bounded, $M(q, \phi)$ is bounded from above and below for all system configurations.

Property II: $F(q, \dot{\phi})$ is a symmetric matrix, which is a consequence of the symmetry of the inertia matrix.

Property III: The matrix $\dot{M}(q, \phi) - 2C(q, \dot{q}, \phi) - F(q, \dot{\phi})$ is skew-symmetric. Notice that the skew-symmetry property for the time-varying case is different from that of the time-invariant case.

Property IV: The dynamic equation (12) is linear in the unknown parameters, i.e.,

$$M(q,\,\phi)\ddot{q}+C(q,\,\dot{q},\,\phi)\dot{q}+F(q,\,\dot{\phi})\dot{q}+g(q,\,\phi) = Y(q,\,\dot{q},\,\ddot{q})\theta(t) \tag{13}$$

where $\theta^T(t) := [\phi^T(t),\,\dot{\phi}^T(t)]$ is the unknown parameter vector and $Y(q,\,\dot{q},\,\ddot{q})$ is the regressor matrix, which depends on the position, velocity, and acceleration in joint space.

III. Adaptive Control

In this section, a passivity-type adaptive controller for the time-varying mechanical system is proposed for tracking a desired trajectory. Let $q_d(t)$ be the desired trajectory. It is assumed that $q_d(t)$ is twice continuously differentiable. Let $e = q(t) - q_d(t)$ be the joint tracking error and $e_{vp} = \dot{e} + \Lambda e$ be the reference velocity error. Consider the control law, τ, given by

$$\tau = Y(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}(t) - K_{vp}e_{vp} \tag{14}$$

where $\dot{q}_r = \dot{q}_d - \Lambda e$, K_{vp}, and Λ are positive definite gain matrices, $\hat{\theta}(t)$ is the estimate of $\theta(t)$, and $Y(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)$ is given by

$$\begin{aligned}Y(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}(t) &= \hat{M}(q,\,\hat{\theta})\ddot{q}_r + \hat{C}(q,\,\dot{q},\,\hat{\theta})\dot{q}_r + \tfrac{1}{2}\hat{F}(q,\,\dot{\hat{\theta}})\dot{q}_r + \tfrac{1}{2}\hat{F}(q,\,\dot{\hat{\theta}})\dot{q}\\ &\quad + \hat{g}(q,\,\hat{\theta})\end{aligned} \tag{15}$$

where $\hat{A}$ represents the estimate of A. Consider the following modification using the linear parameterization property:

$$\begin{aligned}&Y(q,\,\dot{q},\,\ddot{q})\theta\\ &= Y_0(q,\,\dot{q},\,\ddot{q})\theta_0 + Y_1(q,\,\dot{q},\,\ddot{q})\theta_1(t) + Y_2(q,\,\dot{q},\,\ddot{q})\theta_2(t)\end{aligned} \tag{16}$$

where

- θ_0 vector of constant parameters of the system;
- $\theta_1(t)$ vector of time-varying parameters of the system;
- $\theta_2(t)$ vector representing the time rate of change of $\theta_1(t)$, i.e., $\theta_2(t) = d\theta_1(t)/dt$.

Similar to (16), the left-hand-side of (15) is given by

$$\begin{aligned}Y(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta} &= Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}_0 + Y_1(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}_1(t)\\ &\quad + Y_2(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}_2(t).\end{aligned} \tag{17}$$

The time-varying parameter vector $\theta_1(t)$ is parameterized as follows:

$$\theta_1(t) = k_1 f_1(t) + k_2 f_2(t) + \cdots + k_n f_n(t) \tag{18}$$

where $k_1, k_2, \cdots, k_n$ are unknown vector constants and $f_1(t),\, f_2(t),\, \cdots,\, f_n(t)$ are known bounded functions with bounded time derivatives. Hence

$$\dot{\theta}_1(t) = k_1\frac{df_1(t)}{dt} + k_2\frac{df_2(t)}{dt} + \cdots + k_n\frac{df_n(t)}{dt}. \tag{19}$$

A. Closed-Loop Dynamics

Substitute the control law (14) into the dynamic equations (14) to obtain

$$\begin{aligned}&M(q,\,\theta)\ddot{q} + C(q,\,\dot{q},\,\theta)\dot{q} + F(q,\,\dot{\theta})\dot{q} + g(q,\,\theta)\\ &= \hat{M}(q,\,\hat{\theta})\ddot{q}_r + \hat{C}(q,\,\dot{q},\,\hat{\theta})\dot{q}_r + \tfrac{1}{2}\hat{F}(q,\,\hat{\theta}d)\dot{q}_r + \tfrac{1}{2}\hat{F}(q,\,\hat{\theta}d)\dot{q}\\ &\quad + \hat{g}(q,\,\hat{\theta}).\end{aligned} \tag{20}$$

Subtract

$$M(q,\,\theta)\ddot{q}_r + C(q,\,\dot{q},\,\theta)\dot{q}_r + \tfrac{1}{2}F(q,\,\dot{\theta})\dot{q}_r$$

from both sides of (20) and simplify to obtain

$$\begin{aligned}&M(q,\theta)\dot{e}_{vp} + C(q,\dot{q},\theta)e_{vp} + \tfrac{1}{2}F(q,\,\dot{\theta})e_{vp}\\ &= \left[\hat{M}(q,\,\hat{\theta}) - M(q,\,\theta)\right]\ddot{q}_r + \left[\hat{C}(q,\,\dot{q},\,\hat{\theta}) - C(q,\,\dot{q},\,\theta)\right]\dot{q}_r\\ &\quad + \tfrac{1}{2}\left[\hat{F}(q,\,\hat{\theta}d) - F(q,\,\dot{\theta})\right]\dot{q}_r + \tfrac{1}{2}\left[\hat{F}(q,\,\hat{\theta}d) - F(q,\,\dot{\theta})\right]\dot{q}\\ &\quad + \left[\hat{g}(q,\,\hat{\theta}) - g(q,\,\theta)\right].\end{aligned} \tag{21}$$

Define $\tilde{\theta}_0(t) = \hat{\theta}_0(t) - \theta_0$, $\tilde{\theta}_1(t) = \hat{\theta}_1(t) - \theta_1(t)$, $\tilde{\theta}_2(t) = \hat{\theta}_2(t) - \theta_2(t)$. Using the linear parameterization property and (17), (21) can be simplified to

$$\begin{aligned}&M(q)\dot{e}_{vp} + C(q,\,\dot{q})e_{vp} + \tfrac{1}{2}F(q)e_{vp}\\ &= Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\tilde{\theta}_0(t) + Y_1(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\tilde{\theta}_1(t)\\ &\quad + Y_2(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\tilde{\theta}_2.\end{aligned} \tag{22}$$

Since $\theta_1(t) = k_1 f_1(t) + k_2 f_2(t) + \cdots + k_n f_n(t)$, and $\theta_2(t) = d\theta_1(t)/dt$, (22) becomes

$$\begin{aligned}&M(q,\,\theta)\dot{e}_{vp} + C(q,\,\dot{q},\,\theta)e_{vp} + \tfrac{1}{2}F(q,\,\dot{\theta})e_{vp}\\ &= Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\tilde{\phi}_0 + \sum_{i=1}^{n} W_i(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t)\tilde{k}_i\end{aligned} \tag{23}$$

where

$$\begin{aligned}&W_i(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t)\\ &= \left\{f_i(t)Y_1(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r) + Y_2(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\frac{df_i(t)}{dt}\right\}\end{aligned}$$

$\tilde{k}_i(t) := \hat{k}_i(t) - k_i$, and $\hat{k}_i(t)$ is the estimate of k_i.

B. Stability

The following theorem gives the stability of the closed-loop system (23).

Theorem III.1: For the time-varying mechanical system given by (12), the proposed control law (14) with the following update laws for $\theta_0,\, k_1,\, k_2,\, \cdots,\, k_n$:

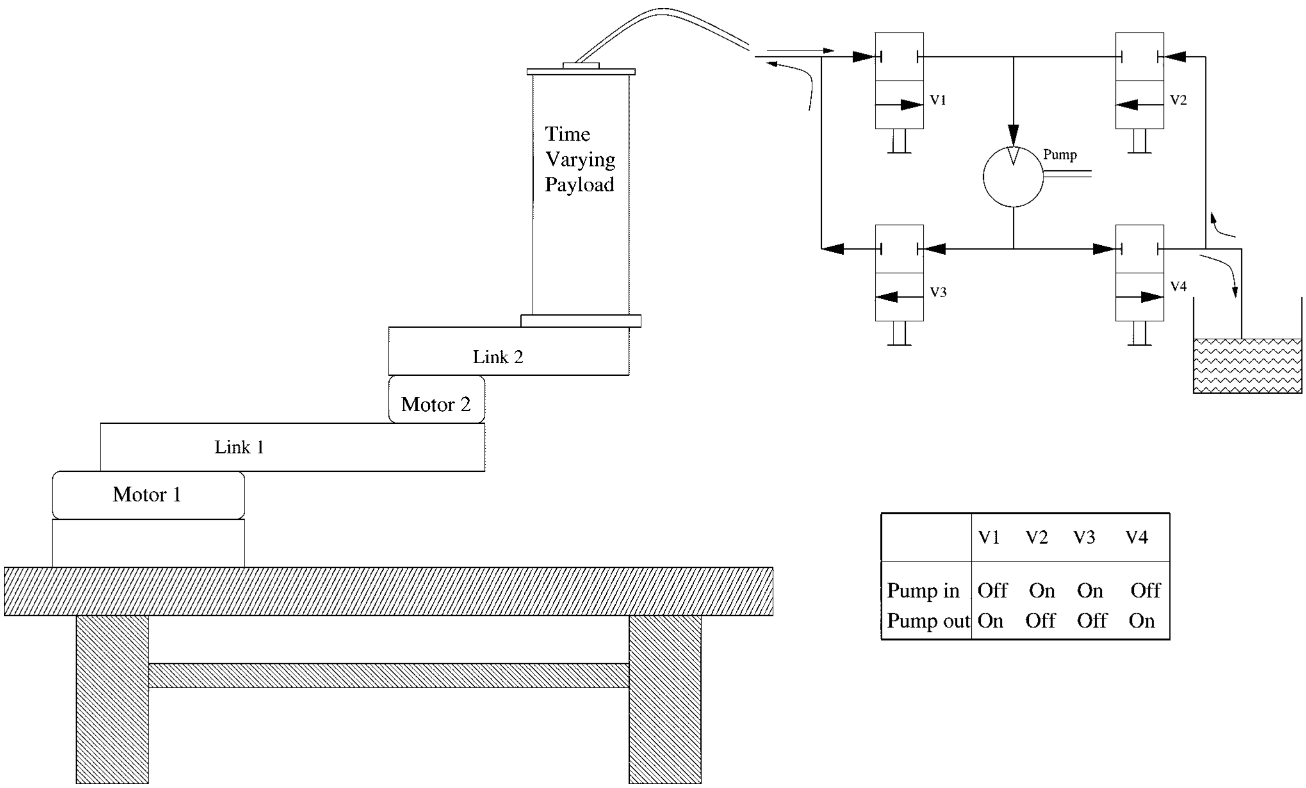
$$\dot{\hat{\theta}}_0 = -\Gamma_0 Y_0^T(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)e_{vp} \tag{24}$$

$$\dot{\hat{k}}_i = -\Gamma_i W_i^T(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t)e_{vp}, \qquad i = 1\,:\,n \tag{25}$$

will result in an asymptotically stable closed-loop system. Γ_0 and Γ_i, $i = 1\,:\,n$, are adaptation gain matrices.

Proof: Consider the following Lyapunov function candidate:

$$V(e_{vp},\,\tilde{\phi}) = \tfrac{1}{2}e_{vp}^T M(q)e_{vp} + \tfrac{1}{2}\tilde{\theta}_0^T\Gamma_0^{-1}\tilde{\theta}_0 + \sum_{i=1}^{n}\tilde{k}_i^T\Gamma_i^{-1}\tilde{k}_i. \tag{26}$$



	V1	V2	V3	V4
Pump in	Off	On	On	Off
Pump out	On	Off	Off	On

Fig. 1. Experimental platform.

Differentiate the Lyapunov function candidate along the trajectories of the closed-loop system (23) and simplify to obtain

$$\begin{aligned}\dot{V} =& \tfrac{1}{2}\, e_{vp}^T \left[\dot{M}(q) - 2C(q,\, \dot{q}) - F(q)\right] e_{vp} - e_{vp}^T K_{vp} e_{vp} \\ &+ e_{vp}^T Y_0(q,\, \dot{q},\, \dot{q}_r,\, \ddot{q}_r)\tilde{\theta}_0 + \dot{\tilde{\theta}}_0^T \Gamma_0^{-1}\tilde{\theta}_0 \\ &+ \sum_{i=1}^{n} e_{vp} W_i(q,\, \dot{q},\, \dot{q}_r,\, \ddot{q}_r,\, t)\tilde{k}_i + \sum_{i=1}^{n} \dot{\tilde{k}}_i^T \Gamma_i^{-1}\tilde{k}_i. \quad (27)\end{aligned}$$

Since $\left[\dot{M}(q) - 2C(q,\, \dot{q}) - F(q)\right]$ is skew-symmetric

$$\begin{aligned}\dot{V} =& -e_{vp}^T K_{vp} e_{vp} + \left[e_{vp}^T Y_0(q,\, \dot{q},\, \dot{q}_r,\, \ddot{q}_r) + \dot{\tilde{\phi}}_0^T \Gamma_0^{-1}\right] \tilde{\phi}_0 \\ &+ \sum_{i=1}^{n} \left\{ e_{vp} W_i(q,\, \dot{q},\, \dot{q}_r,\, \ddot{q}_r,\, t) + \dot{\tilde{k}}_i^T \Gamma_i^{-1} \right\} \tilde{k}_i. \quad (28)\end{aligned}$$

Using the adaptation laws given by (24) and (25) results in

$$\dot{V} = -e_{vp}^T K_{vp} e_{vp}. \quad (29)$$

This implies that e_{vp}, $\tilde{\phi}_0$, $\tilde{k}_i$ are bounded. Further, from (23) $\dot{e}_{vp}$ is bounded. Hence, $\ddot{V}$ is bounded, which implies that $\dot{V}$ is uniformly continuous. Invoking Barbalat's lemma, $\dot{V} \rightarrow 0$ as $t \rightarrow \infty$. Therefore, e_{vp} asymptotically converges to zero. Since $e_{vp} = \dot{e} + \lambda e$, both $e(t)$ and $\dot{e}(t)$ asymptotically converge to zero.

IV. Experiments

The experimental platform consists of a two-degree-of-freedom direct drive manipulator with a cylindrical vessel on the end of the second link. A pipe is connected from the top of vessel to a pump either to pump fluid in or out of the vessel. Pumping of fluid in or out of the vessel during the motion of the robot gives the time-varying nature for the payload. A sketch of the experimental platform is shown in Fig. 1.

The two-degree-freedom manipulator is driven by two direct-drive switched reluctance-type NSK motors. The base motor (model 1410) and elbow motor (model 608) have a maximum rated torque of 245 and 39.2 N·m, respectively. Sensors for both position and velocity measurement are integrated within each motor, which provide measurement of joint position and joint velocity. The actuator position of each link is measured with a 150-pole resolver, which provides a resolution of about 2 arcseconds. The analog position signal is processed through a 10-bit resolver to a digital converter that provides $150 \times 1024 = 153\,600$ counts per resolution. This gives a resolution of 0.000 040 9 radians per encoder counter. A velocity signal is also available through frequency-through-voltage converter that provides an analog signal that is proportional to joint velocity. However, due to noise, joint velocities used in the experiment were calculated from the joint position using a first-order finite difference method.

A servo sampling rate of 4 ms is used in the implementation. Also, "torque mode" is chosen as the operator mode for the NSK motors. Under this mode, the motors behave like current amplifiers which produce a motor torque command that is proportional to the input voltage signal.

The time-varying payload is a cylindrical aluminum vessel of 0.1778 m (7 in) diameter, which is shown in Fig. 1. A thick aluminum mount is built to mount the vessel on the second link of the robot. The length of the vessel is 0.4064 m (16 in). The vessel has an approximate volume of 37.54×10^{-6} m^3 (615 in^3). The working fluid used in our experiment is water and its density is 998.2 kg/m^3 at room temperature. So, the mass of the payload when the vessel is full of water is approximately 10 kg. Hence the time-varying payload can be varied from 0 to 10.0 kg. A garden pump is used to pump water in or out of the vessel. A 0.0095-m (3/8 in) diameter pipe is used to connect the vessel to the pump which provides approximately 0.2 kg/s flow rate. A valve mechanism is designed such that pumping water in or out can be done by switching the inlet and outlet of the pump using the valve mechanism.

A. Dynamic Equations of the Two-Link Robot with Time-Varying Payload

The dynamic equations of the manipulator are given by

$$M(q, \theta)\ddot{q} + C(q, \dot{q}, \theta)\dot{q} + F(q, \dot{\theta})\dot{q} = \tau \tag{30}$$

where $q \in \mathbb{R}^2$ is the joint position vector and $\tau \in \mathbb{R}^2$ are the motor torques. The elements of matrices $M(q, \theta)$, $C(q, \dot{q}, \theta)$, and $F(q, \theta)$ are given as follows:

$$\begin{aligned} M_{11} &= p_1 + 2p_2 \cos(q_2) + v_1(q_2)m_p(t) + I_p(t) \\ M_{12} &= p_3 + p_2 \cos(q_2) + v_2(q_2)m_p(t) + I_p(t) \\ M_{21} &= M_{12} \\ M_{22} &= p_3 + v_3 m_p(t) + I_p(t) \end{aligned}$$

and

$$C(q, \dot{q}, \theta) = -\begin{bmatrix} \dot{q}_2 p_7 & (\dot{q}_1 + \dot{q}_2) p_7 \\ -\dot{q}_1 p_7 & 0 \end{bmatrix}$$

and

$$F(q, \dot{\theta}) = \begin{bmatrix} v_1(q_2) & v_2(q_2) \\ v_2(q_2) & v_3 \end{bmatrix} \dot{m}_p + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \dot{I}_p$$

where p_1, p_2, p_3 are the constant coupled parameters of the robot that contain masses and inertias of the links and the motors, $m_p(t)$ and $I_p(t)$ are the payload mass and inertia, respectively, $p_7(m_p, q) = (p_2 + l_1 l_2 m_p(t)) \sin(q_2)$, $v_1(q) = l_1^2 + l_2^2 + 2l_1 l_2 \cos(q_2)$, $v_2(q) = l_2^2 + l_1 l_2 \cos(q_2)$, and $v_3 = l_2^2$. The robot link lengths are represented by l_1 and l_2. The dynamics is linear in the unknown parameters

$$M(q, \theta)\ddot{q} + C(q, \dot{q}, \theta)\dot{q} + F(q, \dot{\theta})\dot{q} = Y(q, \dot{q}, \ddot{q})\theta \tag{31}$$

Also, $Y(q, \dot{q}, \ddot{q})$ can be decomposed to associate with constant parameters and time-varying parameters as

$$\begin{aligned} &Y(q, \dot{q}, \ddot{q})\theta \\ &\quad = Y_0(q, \dot{q}, \ddot{q})\theta_0 + Y_1(q, \dot{q}, \ddot{q})\theta_1(t) + Y_2(q, \dot{q}, \ddot{q})\theta_2(t) \end{aligned} \tag{32}$$

where $\theta_0^T = [p_1, p_2, p_3]$, $\theta_1(t) = [m_p(t),\ I_p(t)]$ and $\theta_2(t) = [\dot{m}_p(t), \dot{I}_p(t)]^T$. Since the payload is a cylindrical vessel of a fixed diameter, the inertia of the payload $I_p(t) = m_p(t)(R^2/2)$, where R is the radius of the cylindrical vessel. According to the water pump specifications the rated flow rate of the pump is constant within the operating conditions of the pump. This means that the mass of the payload varies according to

$$m_p(t) = k_1 t \tag{33}$$

where k_1 represents the constant water flow rate in or out of the vessel. For the robot dynamics the regressor matrix $Y_0(q, \dot{q}, \ddot{q})$ is given by

$$Y_0(\cdot) = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & 2c_2(\ddot{q}_1 + \ddot{q}_2) - (2\dot{q}_1 + \dot{q}_2^2)s_2 \\ 0 & \ddot{q}_1 + \ddot{q}_2 & c_2\ddot{q}_1 + s_2\dot{q}_1^2 \end{bmatrix} \tag{34}$$

where $s_2 = \sin(q_2)$ and $c_2 = \cos(q_2)$. Define

$$\begin{aligned} y_{11} &:= v_1(q)\ddot{q}_1 + v_2(q)\ddot{q}_2 - l_1 l_2 \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) \\ y_{12} &:= \ddot{q}_1 + \ddot{q}_2 \\ y_{13} &:= v_2(q)\dot{q}_1 + v_3\dot{q}_2 \\ y_{14} &:= \dot{q}_1 + \dot{q}_2 \\ y_{21} &:= v_2(q)\ddot{q}_1 + v_3\ddot{q}_2 + l_1 l_2 \sin(q_2)\dot{q}_1^2 \\ y_{22} &:= y_{12} \\ y_{23} &:= v_2(q)\dot{q}_1 + v_3\dot{q}_2 \\ y_{24} &:= y_{14}. \end{aligned}$$

Then the matrices $Y_1(q, \dot{q}, \ddot{q})$ and $Y_2(q, \dot{q}, \ddot{q}, t)$ are given by

$$Y_1(q, \dot{q}, \ddot{q}) = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}, \quad Y_2(q, \dot{q}, \ddot{q}) = \begin{bmatrix} y_{13} & y_{14} \\ y_{23} & y_{24} \end{bmatrix}. \tag{35}$$

Let $\alpha = R^2/2$. Then since $I_p(t) = \alpha m_p(t)$, we obtain

$$\theta_1(t) = \begin{bmatrix} t \\ t\alpha \end{bmatrix} k_1, \quad \text{and} \quad \theta_2(t) = \begin{bmatrix} 1 \\ \alpha \end{bmatrix} k_1. \tag{36}$$

Therefore

$$\begin{aligned} W_1(q, \dot{q}, \ddot{q}, t)k_1 &:= Y_1(q, \dot{q}, \ddot{q})\theta_1(t) + Y_2(q, \dot{q}, \ddot{q})\theta_2(t) \\ &= \begin{bmatrix} t(y_{11} + \alpha y_{12}) + y_{13} + \alpha y_{14} \\ t(y_{21} + \alpha y_{22}) + y_{23} + \alpha y_{24} \end{bmatrix} k_1. \end{aligned} \tag{37}$$

Since $m_p(t) = k_1 t$, this means that $f_1(t) = t$, according to the parameterization given in (18). The function $f_1(t)$ in this case is unbounded. As a result of this the regressor matrix $W_1(q, \dot{q}, \ddot{q}, t)$ is unbounded. To avoid this problem, a technique of time-scaling is given below.

B. Time Scaling and Adaptive Controller

To circumvent the unboundedness problem for the time-varying payload, each cycle period is mapped into a unit interval, [0, 1]. This is accomplished as follows. Suppose the robot is performing a periodic trajectory with a period T. Then the time-varying payload can be written as

$$m_p(t) = k_1 T \left(\frac{t}{T}\right). \tag{38}$$

Notice that $t/T \in [0, 1]$. Define $k_1' = k_1 T$. Then (37) becomes

$$W_1(q, \dot{q}, \ddot{q}, t)k_1 = \underbrace{\begin{bmatrix} \frac{t}{T}(y_{11} + \alpha y_{12}) + y_{13} + \alpha y_{14} \\ \frac{t}{T}(y_{21} + \alpha y_{22}) + y_{23} + \alpha y_{24} \end{bmatrix}}_{W_1'(q, \dot{q}, \ddot{q}, t/T)} k_1' \tag{39}$$

where $W_1'(q,\,\dot{q},\,\ddot{q},\,t/T)$ represents the scaled version of $W_1(q,\,\dot{q},\,\ddot{q},\,t)$. The new parameter k_1' is estimated instead of k_1. The following is the control law for the two-link robot with a time-varying payload:

$$\tau = Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}_0(t) + W_1'(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t/T)\hat{k}_1'(t) - K_{vp}e_{vp} \tag{40}$$

and the adaptation laws are

$$\dot{\hat{\theta}}_0(t) = -\Gamma_0 Y_0^T(q,\,\dot{q},\,\dot{q}_r,\ddot{q}_r)e_{vp} \tag{41}$$

$$\dot{\hat{k}}_1'(t) = -\Gamma_1 W_1'^T(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t/T)e_{vp} \tag{42}$$

where K_{vp} is the positive definite feedback gain matrix, and Γ_1 is the adaptation gain matrix. Also, $Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)$ is given by

$$Y_0(\cdot) = \begin{bmatrix} \ddot{q}_{r1} & \ddot{q}_{r2} & y_{013} \\ 0 & \ddot{q}_{r1}+\ddot{q}_{r2} & c_2\ddot{q}_{r1}+s_2\dot{q}_1\dot{q}_{r1} \end{bmatrix} \tag{43}$$

where $y_{013} = 2c_2(\ddot{q}_{r1}+\ddot{q}_{r2}) - (\dot{q}_1\dot{q}_{r2}+\dot{q}_{r1}\dot{q}_2+\dot{q}_2\dot{q}_{r2})s_2$, and

$$W_1'(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t/T) = \begin{bmatrix} \frac{t}{T}(z_{11}+\alpha z_{12}) + z_{13} + \alpha z_{14} \\ \frac{t}{T}(z_{21}+\alpha z_{22}) + z_{23} + \alpha z_{24} \end{bmatrix} \tag{44}$$

where

$$\begin{aligned} z_{11} &:= v_1(q)\ddot{q}_{r1} + v_2(q)\ddot{q}_{r2} - l_1 l_2\,\sin(q_2)(\dot{q}_2\dot{q}_{r2}+\dot{q}_1\dot{q}_{r2} + \dot{q}_{r1}\dot{q}_2) \\ z_{12} &:= \ddot{q}_{r1} + \ddot{q}_{r2} \\ z_{13} &:= \tfrac{1}{2}\left(v_2(q)\dot{q}_1 + v_3\dot{q}_2\right) + \tfrac{1}{2}\left(v_2(q)\dot{q}_{r1} + v_3\dot{q}_{r2}\right) \\ z_{14} &:= \tfrac{1}{2}(\dot{q}_1+\dot{q}_2) + \tfrac{1}{2}(\dot{q}_{r1}+\dot{q}_{r2}) \\ z_{21} &:= v_2(q)\ddot{q}_{r1} + v_3\ddot{q}_{r2} + l_1 l_2\,\sin(q_2)\dot{q}_1\dot{q}_{r1} \\ z_{22} &:= z_{12} \\ z_{23} &:= \tfrac{1}{2}\left(v_2(q)\dot{q}_1 + v_3\dot{q}_2\right) + \tfrac{1}{2}\left(v_2(q)\dot{q}_{r1} + v_3\dot{q}_{r2}\right) \\ z_{24} &:= z_{14}. \end{aligned}$$

In the general case, $\hat{k}_i(t)$ and $\hat{k}_i'(t)$ are related by a constant T^i, that is $\hat{k}_i(t) = (1/T^i)\hat{k}_i'(t)$. Also, notice that the adaptive control law (40) is convenient if several cycles of a periodic trajectory are implemented. Since accumulation of the payload mass should be taken into account after every cycle, the control law can be changed to

$$\begin{aligned} \tau(j) &= Y_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\hat{\theta}_0(t) + W_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r)\beta(j-1) \\ &\quad + W_1(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r,\,t)\hat{k}_1'(t) - K_{vp}e_{vp} \\ \beta_j &= \beta_{j-1} + \int_0^1 \hat{k}_{1,\,j-1}'(\omega)\,d\omega \end{aligned} \tag{45}$$

where $\beta(0) = 0$, $\hat{k}_{1,0}' = 0$, and

$$W_0(q,\,\dot{q},\,\dot{q}_r,\,\ddot{q}_r) = \begin{bmatrix} z_{11}+\alpha z_{12} \\ z_{21}+\alpha z_{22} \end{bmatrix}$$

and j denotes the cycle index, that is $j = 1$ represents the first cycle, and so on. Notice that $\beta(j)$ is updated only once at the beginning the cycle. It is the amount of payload added/removed in cycle j to the container, which represents a constant payload for the $(j+1)$th cycle. Control law (45) decouples the constant parameters from the time-varying payload, i.e., the adaptation laws for the constant parameters of the robot are not affected by the adaptation laws of the time-varying payload, and vice-versa. Experimental results clearly validate this aspect.

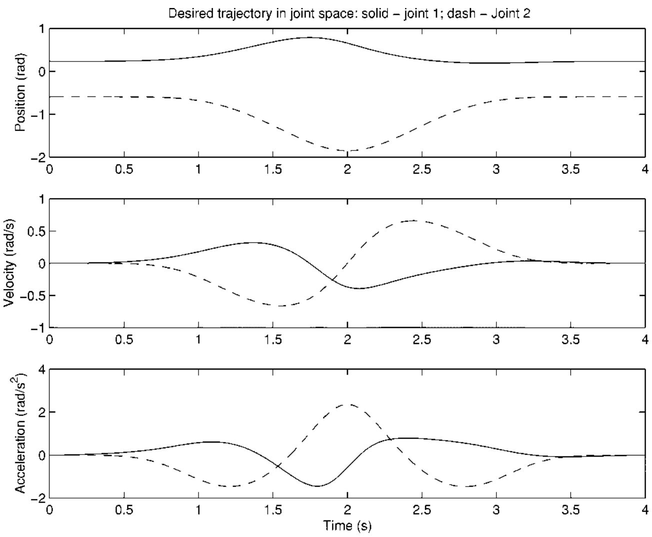


Fig. 2. Desired joint space trajectory.

C. Experimental Results

Extensive experiments were conducted using the proposed adaptive controller. Results from typical experiments are presented here. The desired trajectory for the robot end-effector is a circle with a period of 4 s. The desired joint space trajectories are shown in Fig. 2. Servo sampling rate of 4 ms is used in the experiments. It takes about ten cycles to fill the vessel with water. For safety of not spilling the water during the motion of the robot six cycles for each experiment are used.

Two sets of experiments were conducted: i) pumping water into the cylindrical vessel, i.e., increasing the mass of the payload and ii) emptying water from the vessel, i.e., decreasing the mass of the payload. Similar results were obtained for both pumping water into the vessel and out of the vessel. Figs. 3–8 show the results.

For pumping water into the vessel, the joint errors, the payload estimate, and the robot parameters estimates are shown in Figs. 3–5, respectively. It should be noticed that pumping water into the vessel means that the rate of change of the mass ($\dot{m}_p$) is positive. Figs. 6–8 give results for pumping water out of the vessel. The tracking errors for both pump-in (Fig. 3) and pump-out (Fig. 6) are bounded by about 1/4th of a degree for joint 1 and 1/2 a degree for joint 2. For the pump-in case, the true values of the constant robot parameters without any payload are $p_1 = 3.4$, $p_2 = 0.4$, and $p_3 = 0.3$. For the pump-out case, the vessel is initially filled with water so that the mass of the payload is about 6.0 kg. Therefore, the true values of the robot constant parameters including the payload are $p_1 = 4.8$, $p_2 = 0.9$, and $p_3 = 0.7$. No initial knowledge of k_1 is assumed in both pump-in and pump-out cases. From Figs. 4 and 7, it can be observed that the payload estimate converges to the true value for both pump-in and pump-out cases.

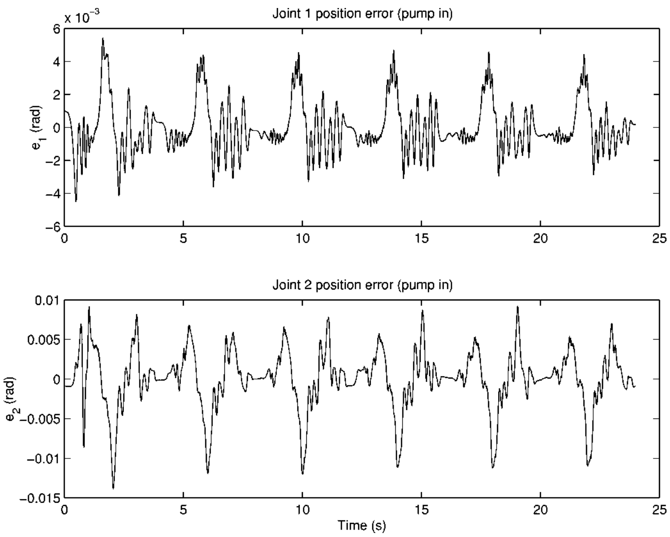


Fig. 3. Joint position errors (pump-in).

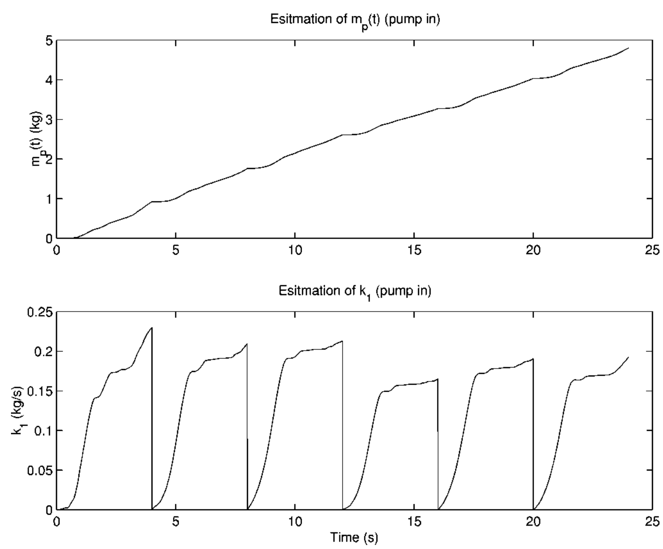


Fig. 4. Payload estimation (pump-in).

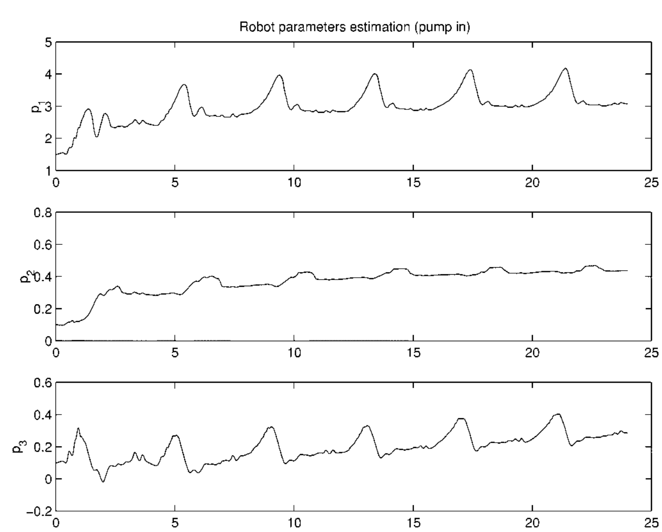


Fig. 5. Robot parameters estimation (pump-in).

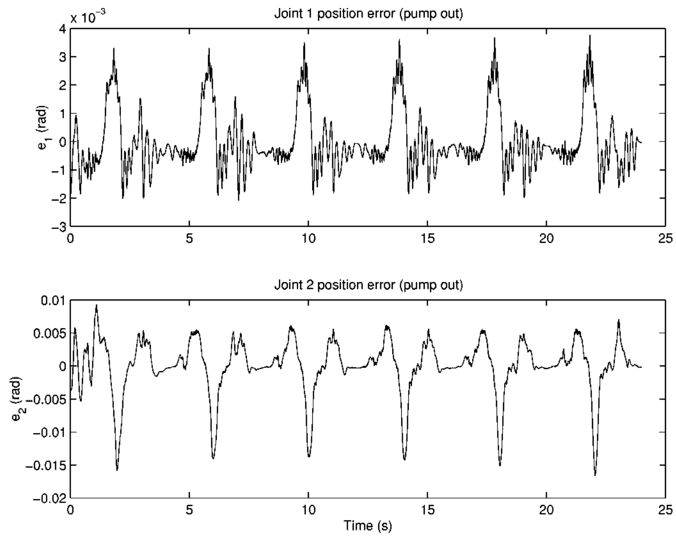


Fig. 6. Joint position errors (pump-out).

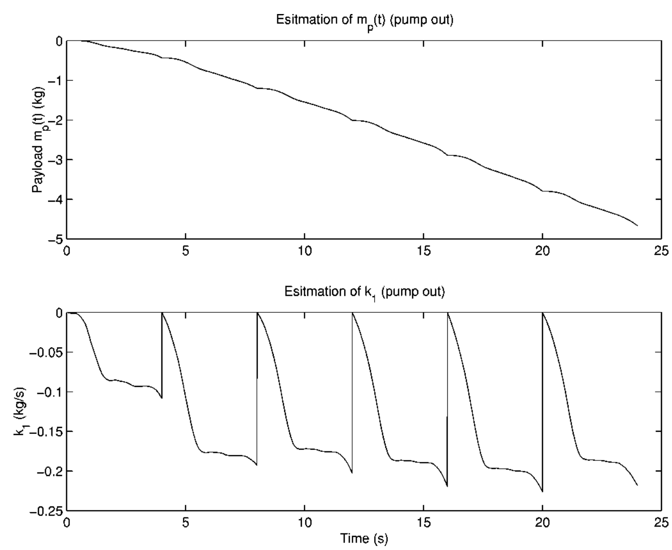


Fig. 7. Payload estimation (pump-out).

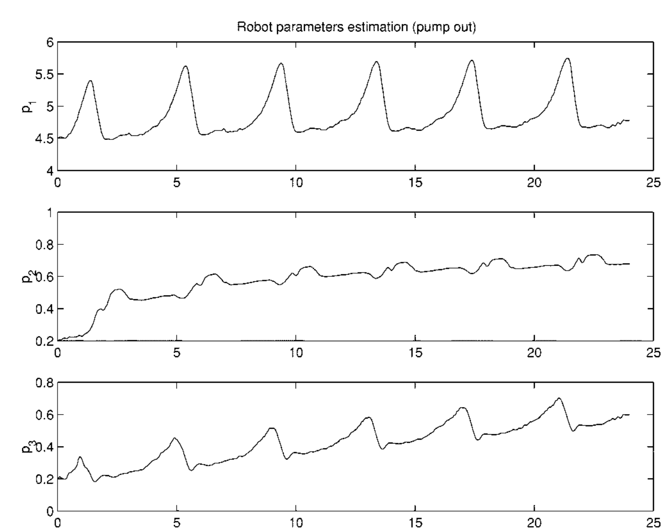


Fig. 8. Robot parameters estimation (pump-out).

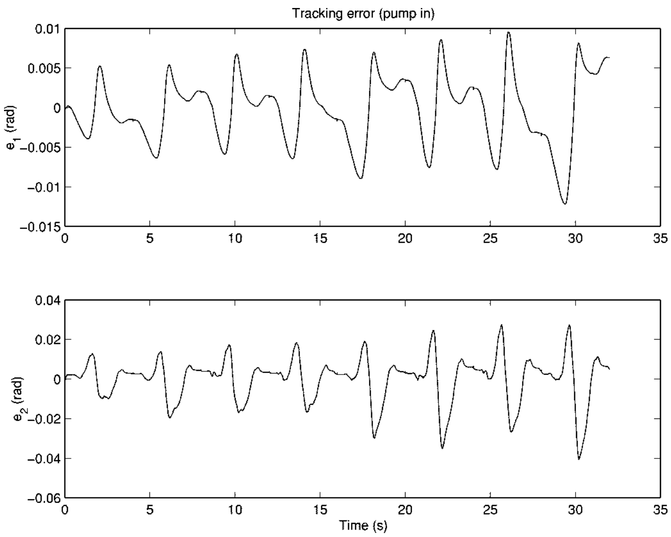


Fig. 9. Joint position errors (pump-in, no adaptation).

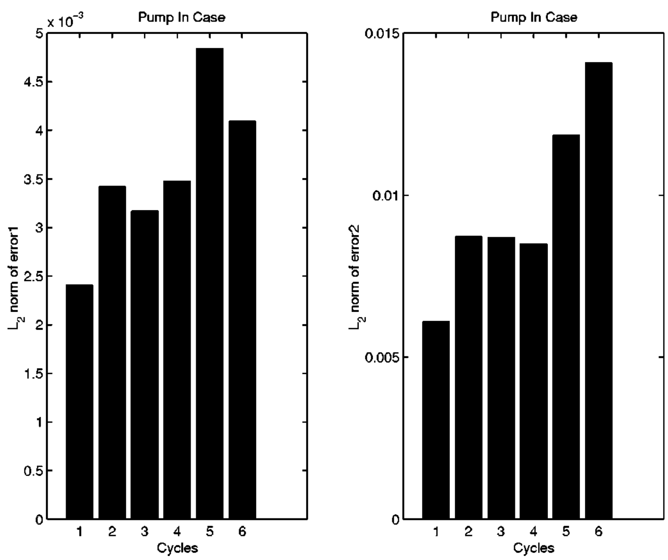


Fig. 10. L_2 norm of joint position errors (pump-in, no adaptation).

Figs. 9 and 10 give the joint tracking errors and L_2 norm of the joint tracking errors for each cycle for a standard passivity-type controller without adaptation. Fig. 11 gives the L_2 norm of the joint tracking errors for the proposed adaptive controller. Comparing Fig. 10 with Fig. 11, it can be observed that the L_2 norm of joint tracking errors keeps increasing after each cycle for a standard passivity-type controller without adaptation, whereas for the proposed controller it keeps decreasing after each cycle.

Remarks:

- Parameter estimates are influenced by low velocity friction at the beginning and the end of each cycle. Also, peaks in constant parameter estimates correspond to peaks in desired acceleration. Also, notice that k_1 estimates are affected at the beginning and end of the cycle. This may be primarily due to the presence of low-velocity friction, since in the middle of the cycle where the velocity is higher, the estimate of k_1 is flat.

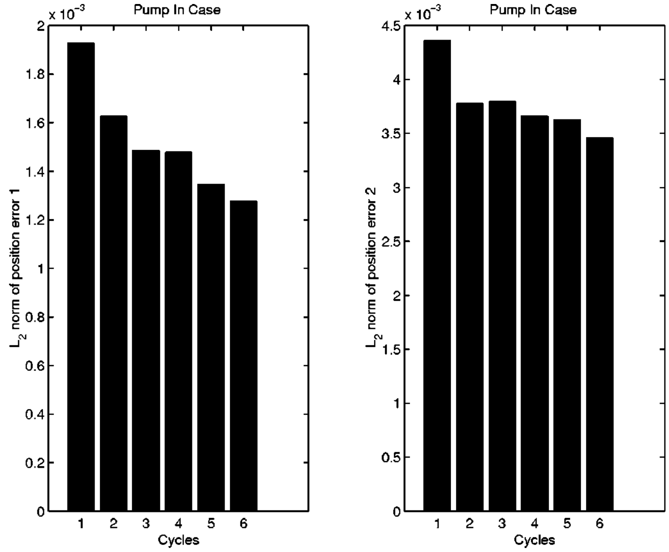


Fig. 11. L_2 norm of joint position errors (pump-in), with adaption.

- Estimation of k_1 is independent of the estimation of the constant robot parameters. We conducted experiments with no payload (container empty) and with full payload (container filled with water). The parameter pattern and the converged values are same as in Figs. 5 and 8.
- The water pump does not provide constant flow rate at all times, and this is reflected in the estimation of the rate of change of the payload, i.e. k_1. Moreover, since the time-scaling maps each cycle period to a unit interval, robustness in payload estimate is obtained; see Figs. 4 and 7.
- Robot parameter estimates are influenced by the desired acceleration whereas the k_1 estimate is influenced by the desired velocity. This is due to the form of the corresponding regressors, i.e., $Y_0(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ given by (43), and $W_1(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t)$ given by (44). Notice that $Y_0(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ has elements involving desired acceleration. The elements in $W_1(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t)$ which have the desired acceleration term are also multiplied by time t.
- Experiments were also conducted with typical robot adaptive control algorithms designed for the constant parameter case, ignoring the time-varying payload mass, hoping that the adaptive estimates for constant parameters will pick up the time-varying payload. This was not the case and the robot went unstable.

V. Conclusions

This work focused on design and implementation of an adaptive control algorithm for time-varying mechanical systems. First, a dynamic model for time-varying mechanical systems is derived based on prior work under the assumption that the generalized constraints are time-invariant but the system parameters such as masses and payloads are time-varying. Based on this dynamic model, an adaptive controller is developed assuming that the time-varying parameters are linearly parameterized by a group of known bounded time functions and unknown constants. Asymptotic stability of the closed-loop system with the proposed adaptive controller is shown. An experimental platform that mimics filling/pouring operations using robot manipulators was designed to test the proposed adaptive controller. Experiments conducted in this paper use a constant rate of change of payload mass. Hence the time-varying payload linearly depends on time and is unbounded. To circumvent this problem a technique of time-scaling, where each cycle of the trajectory is mapped to a unit interval, is proposed. Experimental results validate the effectiveness of the proposed adaptive control design. This work has considered a time-varying payload that is linear in time. Future research will focus on general payloads such as periodic functions.

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