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Adaptive Control of Mechanical Systems With Time-Varying Parameters and Disturbances

A new adaptive control algorithm for mechanical systems with time-varying parameters and/or time-varying disturbances is proposed and investigated. The proposed method does not assume any structure to the time-varying parameter or disturbance. The method is based on the expansion of the time-varying parameter/disturbance using Taylor's formula. This facilitates expanding a time-varying function as a finite length polynomial and a bounded residue. The coefficients of the finite-length polynomial are estimated in a small time interval so that they can be assumed to be constant within that interval. A gradient projection algorithm is used to estimate the parameters within each time interval. Stability of the proposed adaptive controller is shown and discussed. A novel experiment is designed using a two-link planar mechanical manipulator to investigate the proposed algorithm experimentally. Results of the proposed adaptive controller are compared with an ideal nonadaptive controller that assumes complete knowledge of the parameters and disturbances. A representative sample of the experimental results is shown and discussed. [DOI: 10.1115/1.1789538]

1 Introduction

Broadly, adaptive control refers to control of partially known systems. It is common that a control engineer does not know the value of the true parameters of the system being controlled; this has led to continued strong interest in adaptive control research. The amount of adaptive control research for systems with uncertain constant parameters is much larger than systems that have uncertain time-varying parameters. As pointed out in [1], one of the compelling reasons for considering adaptive methods in practical applications is to compensate for large variations in plant parameter values. The focus of this work is on the design of a practical adaptive-control algorithm for time-varying mechanical systems, which are an important practical class of nonlinear systems with time-varying parameters.

High-performance tracking control of mechanical systems is essential in a number of industrial applications; examples include material handling and parts assembly. In many industrial applications, the mechanical system dynamics is time-varying due to a time-varying payload and/or time-varying disturbances. Examples of such applications include pouring and filling operations using robots. There has been an increase in recent research activity in adaptive control of time-varying systems, but most of this research has focused on assuming worst-case bounds for time-varying parameters and/or their derivatives. An amalgam of adaptive and robust control techniques has been used in the control designs with the controller gains chosen based on worst-case bounds. The resulting controllers, although stable, give rise to large and often practically unbounded control inputs.

In [2], a robust switching controller was designed for robot manipulators with time-varying parameters performing path-tracking tasks. Properties of the element-by-element product of matrices was used to isolate the time-varying parameters from the inertia matrix. A robust adaptive controller for robot manipulators consisting of slowly time-varying parameters was presented in [3]. A smooth robust adaptive sliding mode controller was given in [4]. A robust adaptive control algorithm subject to bounded disturbances and bounded and (possibly) time-varying parameters was given in [5]; it was shown that the controller achieves asymptotic tracking if the disturbances vanish and the parameters are constant. In [6], an adaptive controller for time-varying mechanical systems was proposed based on the assumption that the time-varying parameters are given by a group of known bounded time functions and unknown constants. A time-scaling technique of mapping one cycle period of the desired trajectory into a unit interval was proposed to provide robustness to the parameter adaptation algorithms. A novel experimental platform consisting of a two-link manipulator with time-varying payload that mimics filling and pouring operations was built to verify the proposed adaptive algorithm experimentally.

A number of results in adaptive control of linear time-varying plants can be found in [7]. Adaptive control of discrete-time linear systems with time-varying parameters can be found in [8,9]. In [9], the problem of estimating the unknown time-varying parameters is transformed to the problem of observing an unknown state of a linear discrete-time system using Taylor's formula. In [10], it is shown that applying local regression in traditional least-squares algorithm with a forgetting factor can reduce the estimation error in the mean-square sense for systems with slowly time-varying parameters. Regressions techniques and their applications using local polynomial modeling are discussed in great detail in [11].

In this paper, the unknown time-varying parameters and disturbances are assumed to be general unknown time-varying functions. The general time-varying function is expressed as a finite-length polynomial in time and a residue based on Taylor's formula [12]. In the proposed new adaptive controller, the coefficients of the polynomial are estimated in a small time interval so that they can be assumed to be constant, and a robustness term is used in the controller to compensate for the unknown residue. The robustness term is much smaller than what is generally used in the robust control literature and is proportional to the choice of the time interval chosen for estimation. To validate the proposed adaptive controller, an experiment was designed on a two-link manipulator platform. The elbow link of the two-link planar manipulator was used to generate a time-varying disturbance to the base link. A constant torque applied to the elbow link, which acts as a payload to the base link, creates a time-varying inertia for the base link and generates a time-varying disturbance to the base link

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due to the coupling of the dynamics between the links. The proposed adaptive-control algorithm was implemented for tracking a desired trajectory of the base link with time-varying inertia and disturbance.

The contributions of the paper can be summarized by the following: 1) Design of a stable adaptive controller for mechanical systems with time-varying parameters and disturbances using local polynomial approximations, and 2) experimental evaluation of the adaptive controller and its comparison with an ideal nonadaptive controller.

The rest of the paper is organized as follows. Section 2 gives the dynamics of mechanical systems with time-varying parameters and disturbances. Representation of time-varying parameters via local polynomials is discussed in Section 3. Design of an adaptive controller is given in Section 4. Experimental setup, including how to generate time-varying dynamics to the base link, conditions, and results are discussed in Section 5. Conclusions are given in Section 6.

2 Dynamics of Mechanical Systems With Time-Varying Parameters and Disturbances

The dynamics of an n degree-of-freedom mechanical system with time-varying parameters and disturbances [6] is given by

$$M(q,\theta^*)\ddot{q}+C(q,\dot{q},\theta^*)\dot{q}+F(q,\dot{\theta}^*)\dot{q}+g(q,\theta^*)=\tau+d(t) \tag{1}$$

where $q\in\mathbb{R}^n$ is the vector of generalized coordinates, $M(q,\theta^*)\in\mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q},\theta^*)\in\mathbb{R}^{n\times n}$ is the matrix composed of Coriolis and centrifugal terms, $g(q,\theta^*)\in\mathbb{R}^n$ is the gravity vector, $F(q,\dot{\theta}^*)\in\mathbb{R}^{n\times n}$ is a symmetric matrix, which is a consequence of the symmetry of the inertia matrix, $\theta^*\in\mathbb{R}^m$ is the vector of constant and/or time-varying parameters, $\tau\in\mathbb{R}^n$ is the vector of control inputs, and $d(t)\in\mathbb{R}^n$ is the vector of time-varying disturbances. The properties of the dynamic model (1) are given in the following:

Property I. The inertia matrix, $M(q,\theta^*)$, of the time-varying mechanical system is a symmetric positive definite matrix. Assuming $\theta^*(t)$ is bounded, $M(q,\theta^*)$ is bounded from above and below for all system configurations.

Property II. The matrix $\dot{M}(q,\theta^*)-2C(q,\dot{q},\theta^*)-F(q,\dot{\theta}^*)$ is skew symmetric. Notice that the skew-symmetry property for the time-varying case is different from that of the time-invariant case [2,6,13].

Property III. The dynamic model (1) is linear in the unknown parameters, θ^*, $\dot{\theta}^*$; that is,

$$M(q,\theta^*)\ddot{q}+C(q,\dot{q},\theta^*)\dot{q}+F(q,\dot{\theta}^*)\dot{q}+g(q,\theta^*)=Y_1(q,\dot{q},\ddot{q})\,\theta^* + Y_2(q,\dot{q})\,\dot{\theta}^* \tag{2}$$

where $Y_1(q,\dot{q},\ddot{q})$ and $Y_2(q,\dot{q})$ are the regressor matrices corresponding to $\theta^*(t)$ and $\dot{\theta}^*(t)$, respectively.

3 Representation of Time-Varying Functions

To represent a general time-varying function, consider the following result [12]:

Lemma 1. *Let I be an open interval in $\mathbb{R}$, and f be a p-times continuously differentiable function of I into $\mathbb{R}$; then, for any pair of points t_0, t in I*

$$f(t)=f(t_0)+\frac{(t-t_0)}{1!}f^{(1)}(t_0)+\cdots+\frac{(t-t_0)^{p-1}}{(p-1)!}f^{(p-1)}(t_0) + \int_{t_0}^{t}\frac{(t-\xi)^{p-1}}{(p-1)!}f^{(p)}(\xi)d\xi \tag{3}$$

where $f^{(i)}(\cdot)$ denotes the ith derivative of the function $f(\cdot)$.

As a result of Lemma 1, the time-varying function and its time derivative can be represented locally at t_0 as polynomials of time with constant coefficients; that is,

$$f(t)=a_0(t_0)+a_1(t_0)(t-t_0)+\cdots+a_p(t_0)(t-t_0)^{p-1}+\delta_f(t,t_0),$$
$$:=\sum_{i=0}^{p-1}a_i(t_0)(t-t_0)^i+\delta_f(t,t_0),\quad t\in[t_0,t_0+T) \tag{4}$$

$$\dot{f}(t)=\sum_{i=1}^{p-1}ia_i(t_0)(t-t_0)^{i-1}+\dot{\delta}_f(t,t_0),\quad t\in[t_0,t_0+T) \tag{5}$$

where $\delta_f(t,t_0)=\int_{t_0}^{t}(t-\xi)^{p-1}/(p-1)!\,f^{(p)}(\xi)d\xi$, $a_i(t_0)=(1/i!)f^{(i)}(t_0)$, $i=0,\ldots,p-1$, $f^{(i)}(t_0)$ is the ith time derivative evaluated at $t=t_0$, and T is the window length that can be chosen. Assuming that the window is sufficiently small, $\delta_f(t,t_0)$ is negligible. Suppose that the pth derivative of $f(t)$ is bounded, that is, $\sup_t\|f^{(p)}(t)\|\le c_p$, then $\delta_f(t,t_0)$ can be bounded by

$$|\delta_f(t,t_0)|\le\frac{c_p(t-t_0)^p}{p!}\le\frac{c_pT^p}{p!},\quad t\in[t_0,t_0+T) \tag{6}$$

Therefore, it is possible to approximate $f(t)$ closely by choosing either a higher-order polynomial, that is, p large, or a small interval T such that $t-t_0\le T$, or both. If we choose t_0 as a nondecreasing sequence of time instants with each difference between adjacent t_0 not more than T, that is, partition time into segments with the length of each segment not larger than T, then the time-varying function $f(t)$ can be approximated by a number of polynomials of time locally at each t_0 with constant coefficients a_i. Figure 1 illustrates the idea. In Fig. 1, $f_i(t)$, $i=0,1,\ldots,$ locally represents the function $f(t)$ by a polynomial in the ith window. In general, the coefficients a_i between any two intervals are different.

The time derivative of $\delta_f(t,t_0)$ can be obtained by using Leibnitz rule[1] of differentiating an integral with variable limits, and is given by

$$\dot{\delta}_f(t,t_0)=\int_{t_0}^{t}\frac{(t-\xi)^{p-2}}{(p-2)!}f^{(p)}(\xi)d\xi \tag{7}$$

With the knowledge of the bound on $f^{(p)}(t)$, one can obtain a bound on $\dot{\delta}_f(t,t_0)$ as

$$|\dot{\delta}_f(t,t_0)|\le\frac{c_p(t-t_0)^{p-1}}{(p-1)!}\le\frac{c_pT^{p-1}}{(p-1)!},\quad t\in[t_0,t_0+T) \tag{8}$$

Consider the approximation of the function $f(t)$ locally at t_0 and t_r, $t_r\ne t_0$, by

$$f(t)=\sum_{i=0}^{p-1}a_i(t_0)(t-t_0)^i=\sum_{i=0}^{p-1}a_i(t_r)(t-t_r)^i \tag{9}$$

where $a_i(t_r)=(1/i!)f^{(i)}(t_r)$. To express each $a_j(t_r)$ in terms of $a_i(t_0)$, $i=0,\ldots,p-1$, evaluate the jth derivative of (9) at $t=t_r$; notice that one can do this under the assumption that $t_r-t_0\le T$. The jth derivative of (9) is

$$f^{(j)}(t)=\sum_{i=j}^{p-1}a_i(t_0)\frac{i!}{(i-j)!}(t-t_0)^{i-j} \tag{10}$$

[1]Leibnitz rule

$$\frac{d}{dt}\left[\int_{\theta(t)}^{\psi(t)}f(x,t)dx\right]=\int_{\theta(t)}^{\psi(t)}\frac{\partial f(x,t)}{\partial t}dx-\frac{d\theta(t)}{dt}f(\theta(t),t)+\frac{d\psi(t)}{dt}f(\psi(t),t)$$

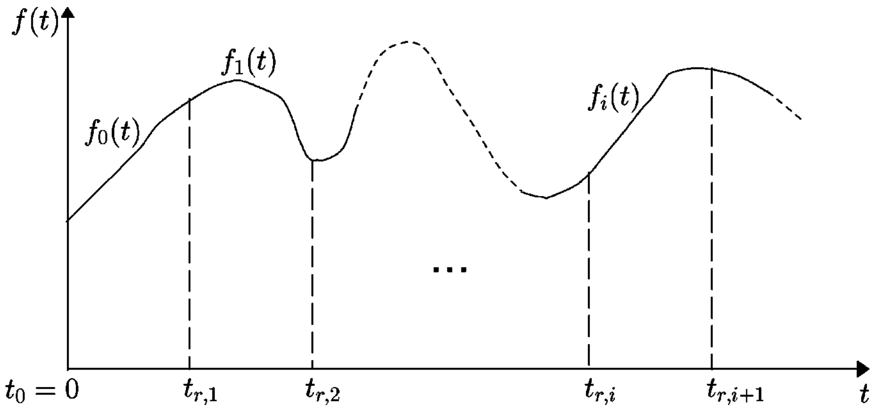

Fig. 1 Local approximation of a continuous function. Each $f_i(t)$ can be approximated by a polynomial in time using Taylor's formula.

$$=\sum_{i=j}^{p-1} a_i(t_r)\frac{i!}{(i-j)!}(t-t_r)^{i-j} \tag{11}$$

Evaluating (10) and (11) at $t=t_r$, we obtain

$$a_j(t_r)=\sum_{i=j}^{p-1} a_i(t_0)\frac{i!}{j!(i-j)!}(t_r-t_0)^{i-j} \tag{12}$$

Therefore, the relationship between $a_j(t_r)$, $j=0,\ldots,p-1$, and $a_i(t_0)$, $i=0,\ldots,p-1$, is given by

$$\begin{bmatrix} a_0(t_r)\\ a_1(t_r)\\ \vdots\\ a_{p-1}(t_r)\end{bmatrix}=\underbrace{\begin{bmatrix}1 & t_r-t_0 & \cdots & (t_r-t_0)^{p-1}\\ 0 & 1 & \cdots & (p-1)(t_r-t_0)^{p-2}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix}}_{A(t_r,t_0)}\begin{bmatrix} a_0(t_0)\\ a_1(t_0)\\ \vdots\\ a_{p-1}(t_0)\end{bmatrix}.$$

$$\times\begin{bmatrix} a_0(t_0)\\ a_1(t_0)\\ \vdots\\ a_{p-1}(t_0)\end{bmatrix} \tag{13}$$

Representing each element of the time-varying parameter vector $\theta^*(t)$ locally at t_0 gives

$$\theta_i^*(t)=\theta_{i0}(t_0)+\theta_{i1}(t_0)(t-t_0)+\cdots+\theta_{i(p-1)}(t_0)(t-t_0)^{p-1}$$
$$+\delta_{\theta_i^*}(t,t_0):=L(t,t_0)\theta_i(t_0)+\delta_{\theta_i^*}(t,t_0) \tag{14}$$

where $\theta_i(t_0):=[\theta_{i0}(t_0),\theta_{i1}(t_0),\cdots,\theta_{i(p-1)}(t_0)]^\mathsf{T}$ is the unknown constant vector and $L(t,t_0):=[1,(t-t_0),\cdots,(t-t_0)^{p-1}]$ is a row vector. Notice that $\theta_i^*(t)$ is the original time-varying parameter that is being approximated by the time-polynomial with coefficients θ_{i0}, θ_{i1}, $\ldots,\theta_{i(p-1)}$. If $t_{r,i}$ is defined as the time instant at which the ith window of the local polynomial approximation begins, then t_0 is given by the sequence $t_0=\{t_{r,i}\}$ with $i=0,1,\ldots$, and $t_{r,i+1}-t_{r,i}=T$. In the following, $t_{r,i}$ is referred to as the resetting time, which is the beginning of the ith window of the local polynomial approximation. Notice that $\theta_i(t_0)$ is constant only within each interval $[t_{r,i},t_{r,i+1})$ and in general differs from one interval to another for a time-varying parameter. The polynomial order $(p-1)$ can be chosen for different $\theta_i^*(t)$ based on some a priori knowledge; for convenience, p is chosen to be the same for all the time-varying parameters. Therefore, the original parameter vector $\theta^*(t)$ is represented by the polynomial coefficient vector $\theta(t_0)$ plus residue vector $\delta_{\theta*}(t,t_0)$ by

$$\theta^*(t,t_0)=\begin{bmatrix}L(t,t_0) & & & \\ & L(t,t_0) & & \\ & & \ddots & \\ & & & L(t,t_0)\end{bmatrix}\theta(t_0)+\delta_{\theta*}(t,t_0)$$
$$:=\Lambda(t,t_0)\theta(t_0)+\delta_{\theta*}(t,t_0) \tag{15}$$

where $\Lambda(t,t_0)$ is an $m\times mp$ matrix, $\theta(t_0):=[\theta_1^\mathsf{T}(t_0),\cdots,\theta_i^\mathsf{T}(t_0),\cdots,\theta_m^\mathsf{T}(t_0)]^\mathsf{T}\in\mathbb{R}^{mp\times 1}$ and $\delta_{\theta*}(t,t_0):=[\delta_{\theta_1^*}(t,t_0),\cdots,\delta_{\theta_i^*}(t,t_0),\cdots,\delta_{\theta_m^*}(t,t_0)]^\mathsf{T}$ is the m-vector consisting of the residue from approximation of each parameter. The time derivative of $\theta^*(t)$ can be represented by

$$\dot{\theta}^*(t,t_0)=\dot{\Lambda}(t,t_0)\,\theta(t_0)+\dot{\delta}_{\theta*}(t,t_0) \tag{16}$$

Since each component of the vectors $\delta_{\theta*}(t,t_0)$ and $\dot{\delta}_{\theta*}(t,t_0)$ is bounded, they are bounded vectors; assume that the bounds are given by

$$\|\delta_{\theta*}(t,t_0)\|\leqslant k_{\delta_{\theta*}}, \quad \forall t\geqslant 0 \tag{17}$$

$$\|\dot{\delta}_{\theta*}(t,t_0)\|\leqslant k_{\dot{\delta}_{\theta*}}, \quad \forall t\geqslant 0 \tag{18}$$

As $\theta(t_0)$ is now a piecewise constant vector, the problem of estimating the time-varying parameter $\theta^*(t)$ in the controller design for (1) can be transformed to that of estimating the constant parameter $\theta(t_0)$ in (15) based on the observations within each interval $[t_0,t_0+T)$. Consequently, various estimation algorithms designed for estimating constant parameters may be employed with appropriate modifications. By using (13), $\theta(t_{r,i+1})$ and $\theta(t_{r,i})$ are related by the following equation:

$$\theta(t_{r,i+1})=\begin{bmatrix} A(t_{r,i+1},t_{r,i}) & & & \\ & A(t_{r,i+1},t_{r,i}) & & \\ & & \ddots & \\ & & & A(t_{r,i+1},t_{r,i}) \end{bmatrix} \times\theta(t_{r,i})$$
$$:=B(t_{r,i+1},t_{r,i})\,\theta(t_{r,i}) \tag{19}$$

Notice that $\theta(t_{r,i})$ is constant in the ith interval, that is, $\theta(\tau)=\theta(t_{r,i})$ for all $\tau\in[t_{r,i},t_{r,i+1}^-]$. Equation (19) will form the basis for resetting the initial value of the estimate at the beginning of each interval. In [9], the same resetting strategy as given by (19) was developed and used in designing an adaptive controller for discrete-time time-varying systems.

In Section 4, the proposed adaptive control algorithm is given, and its stability properties are investigated. A modified gradient projection algorithm is used to estimate the time-varying parameter vector by introducing a resetting scheme at the beginning of each interval; the resetting scheme ensures that the estimate of the time-varying parameter vector, $\hat{\theta}^*(t)$, is continuous, which is consistent with the continuity of the true parameter.

4 Adaptive Control Design

Consider the trajectory tracking problem for the mechanical system (1) with time-varying parameters and disturbances. Let $q_d(t)$ be the desired trajectory. It is assumed that $q_d(t)$ is twice continuously differentiable. Let $e=q(t)-q_d(t)$ be the joint tracking error, and $e_v=\dot{e}+\Gamma e$ be the reference velocity error. The following notations will be used: $\widehat{(*)}$ is the estimate of $(*)$, and $\widetilde{(*)}=\widehat{(*)}-(*)$ is the estimation error of $(*)$. Consider the control law, τ, given by

$$\tau=-K_v e_v+M(q,\hat{\theta}^*)\ddot{q}_r+C(q,\dot{q},\hat{\theta}^*)\dot{q}_r+F(q,\hat{\phi}^*)\frac{\dot{q}+\dot{q}_r}{2} +g(q,\hat{\theta}^*)+\delta_\tau \tag{20}$$

where $\dot{q}_r=\dot{q}_d-\Gamma e$, K_v and Γ are positive definite gain matrices, δ_τ is the additional robust control term, which will be designed later, and

$$\hat{\theta}^*(t,t_0)=\Lambda(t,t_0)\,\hat{\theta}(t_0) \tag{21}$$

$$\hat{\phi}^*(t,t_0)=\dot{\Lambda}(t,t_0)\,\hat{\theta}(t_0) \tag{22}$$

where $\hat{\theta}(t_0)$ will be generated by the adaptation law. Subtracting (15) and (16) from (21) and (22), respectively, results in

$$\tilde{\theta}^*(t,t_0)=\Lambda(t,t_0)\,\tilde{\theta}(t_0)-\delta_{\theta*}(t,t_0) \tag{23}$$

$$\tilde{\phi}^*(t,t_0)=\dot{\Lambda}(t,t_0)\,\tilde{\theta}(t_0)-\dot{\delta}_{\theta*}(t,t_0) \tag{24}$$

where $\tilde{\phi}^*(t,t_0):=\hat{\phi}^*(t,t_0)-\dot{\theta}^*(t,t_0)$. Substitution of the control input (20) into the dynamic equation (1) and simplifying using the linear parametrization property, Property IV, we obtain the error dynamics in terms of e_v as

$$\begin{aligned} M(q,\theta^*)\dot{e}_v+C(q,\dot{q},\theta^*)e_v+\frac{1}{2}F(q,\dot{\theta}^*)e_v+K_v e_v \\ =M(q,\tilde{\theta}^*)\ddot{q}_r+C(q,\dot{q},\tilde{\theta}^*)\dot{q}_r+F(q,\tilde{\phi}^*)\frac{\dot{q}+\dot{q}_r}{2} \\ +g(q,\tilde{\theta}^*)+\delta_\tau+d(t) \\ =Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta}^*+Y_2(q,\dot{q},\dot{q}_r)\tilde{\phi}^*+\delta_\tau+d(t) \end{aligned} \tag{25}$$

where

$$Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta}^*=M(q,\tilde{\theta}^*)\ddot{q}_r+C(q,\dot{q},\tilde{\theta}^*)\dot{q}_r+g(q,\tilde{\theta}^*) \tag{26}$$

$$Y_2(q,\dot{q},\dot{q}_r)\tilde{\phi}^*=F(q,\tilde{\phi}^*)\frac{\dot{q}+\dot{q}_r}{2} \tag{27}$$

Substituting $\tilde{\theta}^*(t,t_0)$ and $\tilde{\phi}^*(t,t_0)$ given by (23) and (24), respectively, into (25) yields

$$\begin{aligned} M(q,\theta^*)\dot{e}_v+C(q,\dot{q},\theta^*)e_v+\tfrac{1}{2}F(q,\dot{\theta}^*)e_v+K_v e_v \\ =Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\Lambda(t,t_0)\tilde{\theta}(t_0)-\delta_{\theta*}(t,t_0))+\delta_\tau+d(t) \\ +Y_2(q,\dot{q},\dot{q}_r)(\dot{\Lambda}(t,t_0)\tilde{\theta}(t_0)-\dot{\delta}_{\theta*}(t,t_0)) \\ =Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta}(t_0)+\delta_\tau-Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\delta_{\theta*}(t,t_0) \\ -Y_2(q,\dot{q},\dot{q}_r)\dot{\delta}_{\theta*}(t,t_0)+d(t) \end{aligned} \tag{28}$$

where $Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)=Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Lambda(t,t_0)+Y_2(q,\dot{q},\dot{q}_r)\dot{\Lambda}(t,t_0)$

In the following, for brevity, all the arguments of vectors and matrices are omitted whenever there is no confusion. Consider the following Lyapunov function candidate during each interval, that is, $t\in[t_{r,i},t_{r,i+1})$,

$$V=\tfrac{1}{2}e_v^{\mathsf{T}}M(q,\theta^*)e_v+\tfrac{1}{2}\tilde{\theta}^{\mathsf{T}}\Gamma_1^{-1}\tilde{\theta} \tag{29}$$

where $\Gamma_1=\Gamma_1^{\mathsf{T}}>0$. The time derivative of V along the trajectories of (28) is

$$\begin{aligned} \dot{V}=e_v^{\mathsf{T}}M(q,\theta^*)\dot{e}_v+\tfrac{1}{2}e_v^{\mathsf{T}}\dot{M}(q,\theta^*)e_v+\tilde{\theta}^{\mathsf{T}}\Gamma_1^{-1}\dot{\tilde{\theta}} \\ =-e_v^{\mathsf{T}}K_v e_v+e_v^{\mathsf{T}}Y\tilde{\theta}+e_v^{\mathsf{T}}(\delta_\tau-Y_1\delta_{\theta*}-Y_2\dot{\delta}_{\theta*}+d) \\ +\tilde{\theta}^{\mathsf{T}}\Gamma_1^{-1}\dot{\tilde{\theta}} \end{aligned} \tag{30}$$

where the Property III is applied.

To estimate the unknown parameter vector $\hat{\theta}$, we use the gradient projection algorithm given in Ref. [14], which we briefly illustrate in the following. Consider a convex parameter set Π given by

$$\hat{\theta}=[\hat{\theta}_1,\cdots,\hat{\theta}_i,\cdots,\hat{\theta}_{mp}]^{\mathsf{T}}\in\Pi\Leftrightarrow|\hat{\theta}_i-\rho_i|<\sigma_i, \quad \forall i\in\{1,mp\} \tag{31}$$

with ρ_i and σ_i some given real numbers. Consider the function

$$\mathcal{P}(\hat{\theta})=\frac{2}{\varepsilon}\left[\sum_{i=1}^{mp}\left|\frac{\hat{\theta}_i-\rho_i}{\sigma_i}\right|^q-1+\varepsilon\right] \tag{32}$$

where $0<\varepsilon<1$ and $q\geqslant 2$. Now, consider the "smooth projection" *Proj*, which will be used to estimate $\hat{\theta}$ while maintaining it in Π:

$$Proj(\hat{\theta},y)=\begin{cases} y, & \text{if } \mathcal{P}(\hat{\theta})<0. \\ y, & \text{if } \mathcal{P}(\hat{\theta})=0 \quad \text{and} \quad \nabla_{\mathcal{P}}^{\mathsf{T}}y\leq 0 \\ y-\dfrac{\mathcal{P}(\hat{\theta})\nabla_{\mathcal{P}}\nabla_{\mathcal{P}}^{\mathsf{T}}}{\|\nabla_{\mathcal{P}}\|^2}y, & \text{otherwise.} \end{cases} \tag{33}$$

where $\nabla_{\mathcal{P}}=[\partial\mathcal{P}(\hat{\theta})/\partial\hat{\theta}]^{\mathsf{T}}$ is a column vector. Based on the smooth projection defined above, $\hat{\theta}$ is estimated by

$$\dot{\hat{\theta}}=\Gamma_1 Proj(\hat{\theta},-Y^{\mathsf{T}}e_v) \tag{34}$$

With the projection algorithm given by (34), we have

$$e_v^{\mathsf{T}}Y\tilde{\theta}+\tilde{\theta}^{\mathsf{T}}\Gamma_1^{-1}\dot{\tilde{\theta}}=\tilde{\theta}^{\mathsf{T}}(Y^{\mathsf{T}}e_v+Proj(\hat{\theta},-Y^{\mathsf{T}}e_v))\leq 0 \tag{35}$$

Substituting (35) into (30) results in

$$\dot{V}\leq -e_v^{\mathsf{T}}K_v e_v+e_v^{\mathsf{T}}(\delta_\tau-Y_1\delta_{\theta*}-Y_2\dot{\delta}_{\theta*}+d) \tag{36}$$

Notice that the Lyapunov function candidate (29) and the adaptation law (34) are designed for each time interval; that is, $t\in[t_{r,i},t_{r,i+1})$. At the beginning of each interval, say $(i+1)$th interval, t_0 is changed from $t_0=t_{r,i}:=iT$ to $t_0=t_{r,i+1}:=(i+1)T$. The matrix $\Lambda(t,t_0)$ will change at this time instant. As a result, from (21) and (22), $\hat{\theta}^*(t,t_0)$ and $\hat{\phi}^*(t,t_0)$ will also change if $\hat{\theta}(t,t_0)$ is not modified. Hence, $\hat{\theta}(t,t_0)$ must be reset at the beginning of each time interval to ensure that $\hat{\theta}^*(t,t_0)$ and $\hat{\phi}^*(t,t_0)$ are continuous. At the beginning of $(i+1)$th time interval, the initial value of the estimate is reset according to the following:

$$\hat{\theta}(t_{r,i+1},t_{r,i+1})=B(t_{r,i+1},t_{r,i})\hat{\theta}(t_{r,i+1}^-,t_{r,i}) \tag{37}$$

The resetting scheme (37) guarantees the continuity of $\hat{\theta}^*(t,t_0)$ and $\hat{\phi}^*(t,t_0)$ at the resetting points $t=t_{r,i+1}$. This is shown in the following. Just before resetting for the $(i+1)$th interval, using (21) for the estimate, we obtain

$$\hat{\theta}^*(t_{r,i+1}^-,t_{r,i})=\Lambda(t_{r,i+1}^-,t_{r,i})\hat{\theta}(t_{r,i+1}^-,t_{r,i}) \tag{38}$$

At the resetting point, again using (21) for the estimate with $t_0=t_{r,i+1}$,

$$\hat{\theta}^*(t_{r,i+1},t_{r,i+1})=\Lambda(t_{r,i+1},t_{r,i+1})\hat{\theta}(t_{r,i+1},t_{r,i+1}) \tag{39}$$

Therefore,

$$\begin{aligned}\hat{\theta}^*(t_{r,i+1},t_{r,i+1})&=\Lambda(t_{r,i+1},t_{r,i+1})B(t_{r,i+1},t_{r,i})\hat{\theta}(t_{r,i+1}^-,t_{r,i})\\ &=\Lambda(t_{r,i+1},t_{r,i})\hat{\theta}(t_{r,i+1}^-,t_{r,i}).\end{aligned} \tag{40}$$

From (38) and (40), since $\Lambda(t,t_0)$ is a continuous function of t, notice that $\hat{\theta}^*(t_{r,i+1}^-,t_{r,i})=\hat{\theta}^*(t_{r,i+1},t_{r,i+1})$. Following along the same lines, we can also show that $\hat{\phi}^*(t_{r,i+1}^-,t_{r,i})=\hat{\phi}^*(t_{r,i+1},t_{r,i+1})$.

The additional robust control term, δ_τ, in (36) is chosen as follows:

$$\delta_\tau=\begin{cases} -(k_{\delta_{\theta*}}\|Y_1\|+k_{\delta_{\theta*}}\|Y_2\|+k_d)\dfrac{e_v}{\|e_v\|}, & \text{if } \|e_v\|\geq\varepsilon_0 \\ -\dfrac{1}{\varepsilon_0}(k_{\delta_{\theta*}}\|Y_1\|+k_{\delta_{\theta*}}\|Y_2\|+k_d)e_v, & \text{if } \|e_v\|<\varepsilon_0 \end{cases} \tag{41}$$

where $\varepsilon_0>0$ and $k_d=\sup_{t\geq 0}d(t)$. It can be shown that the system (28) is uniformly ultimately bounded [15], and e_v converges in finite time to the set Π_1 defined by

$$\Pi_1:=\{e_v:\|e_v\|\leq\varepsilon_0\} \tag{42}$$

Since $e_v(t)$ is bounded and $e_v=\dot{e}+\Gamma e$, the tracking error, $e(t)$, and its time derivative, $\dot{e}(t)$, are also uniformly ultimately bounded. Therefore, $q(t)$, $\dot{q}(t)$, $\dot{q}_r(t)$, and $\ddot{q}_r(t)$ are bounded, since $e(t)$, $\dot{e}(t)$, $q_d(t)$, $\dot{q}_d(t)$, and $\ddot{q}_d(t)$ are bounded. The estimated parameters $\hat{\theta}^*(t)$ and $\hat{\phi}^*(t)$ are also bounded because $\hat{\theta}(t)$ and $\Lambda(t,t_0)$ are bounded. From (20), the control input $\tau(t)$ is bounded as it is composed of all bounded signals. The following theorem summarizes the results of the analysis.

Theorem 4.1. *For the time-varying mechanical system given by* (1), *the proposed adaptive control law given by* (20), *the parameter estimation algorithms given by* (34), *the resetting scheme given by* (37), *and with the knowledge of the bounds given in* (17) *and* (18), *the control input* $\tau(t)$, *the estimated time-varying parameters* $\hat{\theta}^*(t)$ *and* $\dot{\hat{\theta}}^*(t)$, *and the tracking error* $e(t)$ *are uniformly ultimately bounded.*

Remark 1. *In the "ideal" case, that is, the unknown parameter vector,* $\theta^*(t)$, *is constant and the disturbance* $d(t)=0$, *we have* $\delta_{\theta*}=0$, $\dot{\delta}_{\theta*}=0$. *The time derivative of the Lyapunov function candidate given by* (36) *becomes*

$$\dot{V}\leq -e_v^{\mathsf{T}}K_v e_v+e_v^{\mathsf{T}}\delta_\tau=-e_v^{\mathsf{T}}K_v e_v\leq 0 \tag{43}$$

Therefore, asymptotic convergence of $e(t)$ *to zero is achieved. Thus, the proposed adaptive algorithm can be applied to control of mechanical systems irrespective of whether they involve time-varying parameters or not.*

Remark 2. *The disturbance vector* $d(t)$ *can also be approximated locally by polynomials of time. The control input* $\tau(t)$ *is in the same form as* (20) *except that* k_d *in* (41) *is replaced by the upper bound of the approximation error* $\delta_d(t,t_0)$ *given by the following equation*:

$$d(t)=\Lambda'(t,t_0)\theta_d(t_0)+\delta_d(t,t_0)$$

where $\theta_d(t_0)$ *is the coefficient vector and* $\Lambda'(t,t_0)$ *is the matrix that depends on the time interval for approximation. The vector* $\theta_d(t_0)$ *can be estimated in each interval.*

5 Experiments

To experimentally investigate the proposed control algorithm, a time-varying experiment is designed for a two-link robot, which consists of a two-axis direct-drive manipulator as shown in Fig. 2. The direct-drive manipulator operates in the absence of the undesirable factors of mechanical backlash and gear train compliance. Each axis of the manipulator is driven by an NSK Megatorque direct drive servomotor. The NSK-Megatorque motor system consists of a high-torque direct-drive brushless actuator, a high-resolution brushless resolver, and a heavy-duty precision bearing. The servomotors are capable of up to three revolutions per second maximum velocity and position feedback resolution of up to 156,400 counts per revolution. The base motor delivers up to 240 N-m of rated torque output, and the elbow motor produces up to 40 N-m rated torque output. The real-time system associated with the direct drive manipulator consists of a host computer, a servo-DSP card, and a DSP associated with the sensors. For a complete description of the experimental platform we refer the reader to Ref. [6].

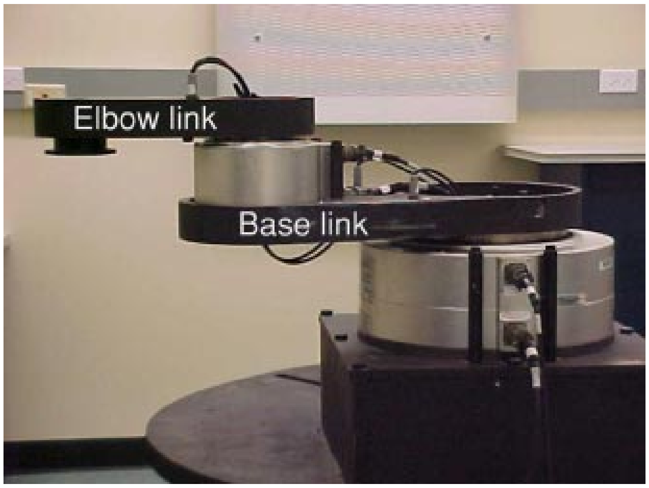

Fig. 2 Picture of the two-link robot

The elbow link of the planar manipulator is used to generate a time-varying disturbance to the base link. This is done as follows. A constant torque is applied to the elbow link. This has an effect of generating a time-varying payload to the base link; that is, due to the rotation of the elbow link, the mass moment of inertia of the base link is varying with time. Further, since the dynamics of both the links is coupled, the motion of the elbow link also causes a time-varying disturbance to the base link. Then, the goal is to control the base link, which has a time-varying inertia and is acted on by time-varying disturbances by using the proposed adaptive controller. The procedure of obtaining the time-varying dynamics for the base link is explained in the following section.

5.1 Generation of Time-Varying Dynamics for the Base Link. The dynamics of the two-link manipulator is given by

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=\tau-f_f \tag{44}$$

where

$$M(q)=\begin{bmatrix} p_1+2p_3c_2 & p_2+p_3c_2 \\ p_2+p_3c_2 & p_2 \end{bmatrix},$$

$$C(q,\dot{q})=\begin{bmatrix} -p_3\dot{q}_2s_2 & -p_3(\dot{q}_1+\dot{q}_2)s_2 \\ p_3\dot{q}_1s_2 & 0 \end{bmatrix}$$

q_1 and q_2 are angular positions of the base and the elbow link, respectively, $\tau=[\tau_1,\tau_2]^{\mathsf{T}}$ is the vector of motor torques, $f_f=[f_1,f_2]^{\mathsf{T}}$ is the vector of friction torques, $c_2=\cos(q_2)$ and $s_2=\sin(q_2)$, and p_1, p_2 and p_3 are coupled inertial parameters. The true values of the coupled inertial parameters without any payload on the elbow link are $p_1=3.4$, $p_2=0.2$ and $p_3=0.15$.

Reducing the two second-order equations given by (44) into a single equation results in

$$(p_1p_2-p_2^2-p_3^2c_2^2)\ddot{q}_1-p_3(2p_2\dot{q}_1\dot{q}_2+p_2\dot{q}_1^2+p_2\dot{q}_2^2+p_3c_2\dot{q}_1^2)s_2$$
$$=p_2(\tau_1-f_1)-(p_2+p_3c_2)(\tau_2-f_2) \tag{45}$$

Equation (45) can be rewritten as

$$I(t)\ddot{q}_1+\dot{I}(t)\dot{q}_1+f_1=\tau_1+d(t) \tag{46}$$

where

$$I(t)=p_1-p_2-\frac{p_3^2}{p_2}c_2^2 \tag{47}$$

$$d(t)=p_3\left((\dot{q}_1+\dot{q}_2)^2+\frac{p_3}{p_2}c_2\dot{q}_1^2+\frac{2p_3}{p_2}c_2\dot{q}_1\dot{q}_2\right)s_2$$
$$-\left(1+\frac{p_3}{p_2}c_2\right)(\tau_2-f_2) \tag{48}$$

$$f_1=f_v\dot{q}_1+f_c\,\mathrm{sgn}(\dot{q}_1) \tag{49}$$

where f_c and f_v are the Coulomb and viscous friction coefficients, respectively. Equation (46) represents the dynamics of a single degree-of-freedom system with time-varying inertia, $I(t)$, and time-varying disturbance, $d(t)$. By choosing τ_2, one can introduce a desired $I(t)$ and $d(t)$. In practice, due to the coupling between the base link and the elbow link, the motion of the base link affects the motion of the elbow link, and consequently affects $I(t)$ and $d(t)$. However, a high constant torque applied to the elbow link will generate a high-velocity, almost constant, rotation of the elbow link; then, the effect of the motion of the base link on $I(t)$ and $d(t)$ is relatively small, and thus can be neglected.

5.2 Experimental Conditions. The desired trajectory for the angular position of the base link is chosen to be sinusoidal with an amplitude of 0.5 radians and a frequency of 0.5 Hz; that is, $q_{d1}(t)=0.5\sin(\pi t)$. The elbow link is used to generate a time-varying disturbance and time-varying moment of inertia to the base link. Data from two sets of experiments is shown in this paper. A constant torque of 4 N-m for the elbow link is used as input in the first case, and a constant torque of 3 N-m is used in the second case. With the applied torques of 4 N-m and 3 N-m, the elbow link will rotate with an angular velocity of around 20 rad/s and 6 rad/s, respectively, after reaching the steady state. A control sampling period of 2 milliseconds is chosen in all the experiments.

To track the desired trajectory, the torque input to the base link, τ_1, is designed using the proposed adaptive controller (20). The parameters $I(t)$, $d(t)$, f_c, and f_v are estimated by $\hat{I}_0+(t-t_0)\hat{I}_1$, $\hat{d}_0+(t-t_0)\hat{d}_1$, $\hat{f}_c$, and $\hat{f}_v$, respectively. Hence, the parameter vector, which is estimated in the experiment, is $\theta^{\mathsf{T}}=[I_0,I_1,d_0,d_1,f_v,f_c]$. The window width for local polynomial approximation is chosen to be 0.1 s; that is, $T=0.1$ s. The gain values used in the experiments are $\Gamma=50$, $K_v=100$, $\Gamma_1=\mathrm{diag}(20,20,100,1000,5,10)$. The constants in the robust control term δ_τ are chosen to be $k_{\delta_{\theta*}}=0.05$, $k_{\delta_{\theta*}}=16$, $k_d=20$, and $\epsilon_0=0.1$. The initial values for the estimate vector $\hat{\theta}$ is chosen to be $\hat{\theta}(0)=[3.4,0,0,0,0,0]^{\mathsf{T}}$. The following bounds for the estimated parameters are chosen in the projection algorithm: $\hat{I}_0\in[1,10]$, $\hat{I}_1\in[-10,10]$, $\hat{d}_0\in[-100,100]$, $\hat{d}_1\in[-2000,2000]$.

5.3 Experimental Results. The data shown in all the figures corresponds to $u_2(t)=4$ N-m during the first 16 s and $u_2(t)=3$ N-m for the remaining 14 s; see bottom plot of Fig. 5. Also, notice that the cycle time of the desired angular position trajectory of the base link is 2 s; therefore, the data correspond to implementation results for 15 cycles.

The time-varying inertia and the disturbance of the base link which are computed by using (47) and (48) are shown in Fig. 3. Notice that the time-varying disturbance is periodic with an amplitude of about 50 N-m (with $u_2=4$ N-m). The moment of inertia is periodic with an average value of 3.15 Kg-m^2 and a peak-to-peak variation of 0.11 Kg-m^2.

The tracking error of the base link is shown in the top plot of Fig. 4. It can be observed that the peak tracking error of the base link is less than 0.04 radians even in the presence of time-varying inertia and very large time-varying disturbance; from 16 s onward, when the variation of the inertia and the disturbance are reduced, the tracking error is also reduced. Notice that the motor torque input of the base link, shown as top plot in Fig. 5, has similar amplitude and frequency as that of the time-varying disturbance. The estimated $d(t)$ and $I(t)$ are shown in Figs. 6 and 7, respectively. Figure 8 shows the estimates of the friction coefficients f_v

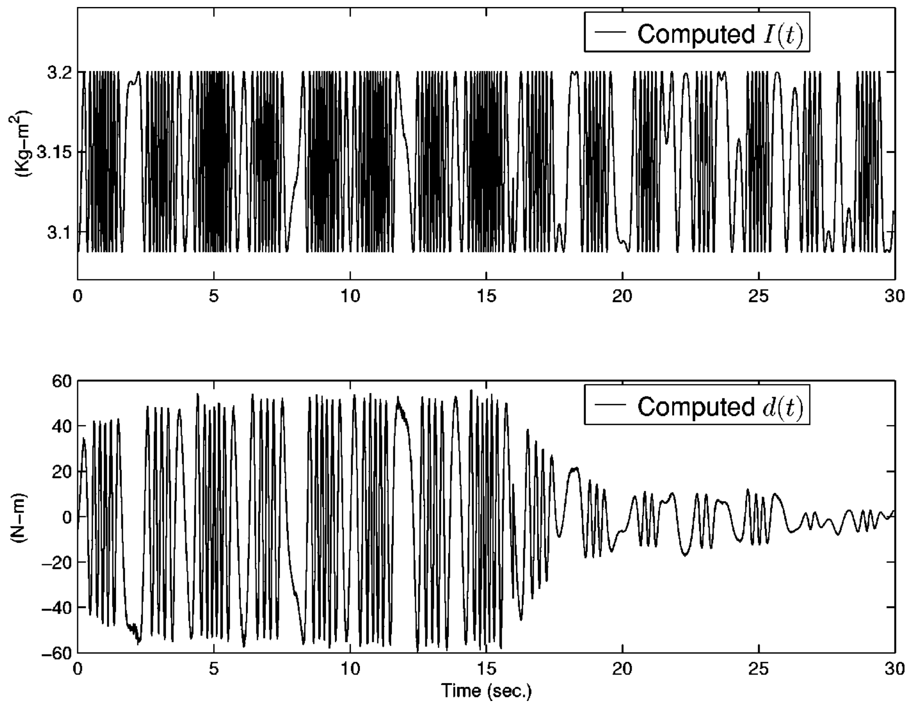

Fig. 3 The time-varying inertia, $I(t)$ (top plot), and the time-varying disturbance, $d(t)$ (bottom plot) are shown. $I(t)$ and $d(t)$ are computed by using the experimental data of $q_2(t)$, $\dot{q}_2(t)$, $\dot{q}_1(t)$ in (47) and (48). The data from zero to 16 s corresponds to $\tau_2 = 4$ N-m and the data from 16 to 30 s corresponds to $\tau_2 = 3$ N-m.

and f_c. It can be observed that all the estimated parameters are within the range defined in the projection algorithm.

5.4 Comparison With an Ideal Nonadaptive Controller. To compare the performance of the proposed adaptive controller with a controller that uses true parameter values, an ideal nonadaptive controller is designed and implemented on the experimental platform. Experimental results of the two controllers are compared and discussed.

Equation (46) can be rewritten in terms of the tracking error of the base link, $e_1 := q_1 - q_{d1}$, as follows:

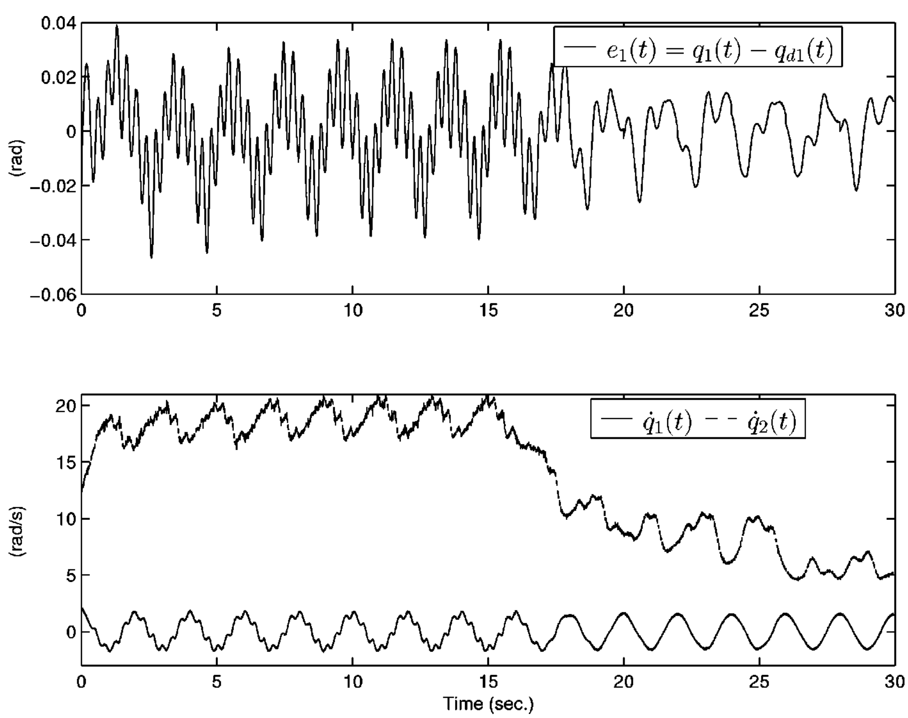

Fig. 4 Tracking error of the base link [$e_1(t)$, top plot] and the angular velocities of the base link and elbow link [$\dot{q}_1(t)$ and $\dot{q}_2(t)$, bottom plot] are shown

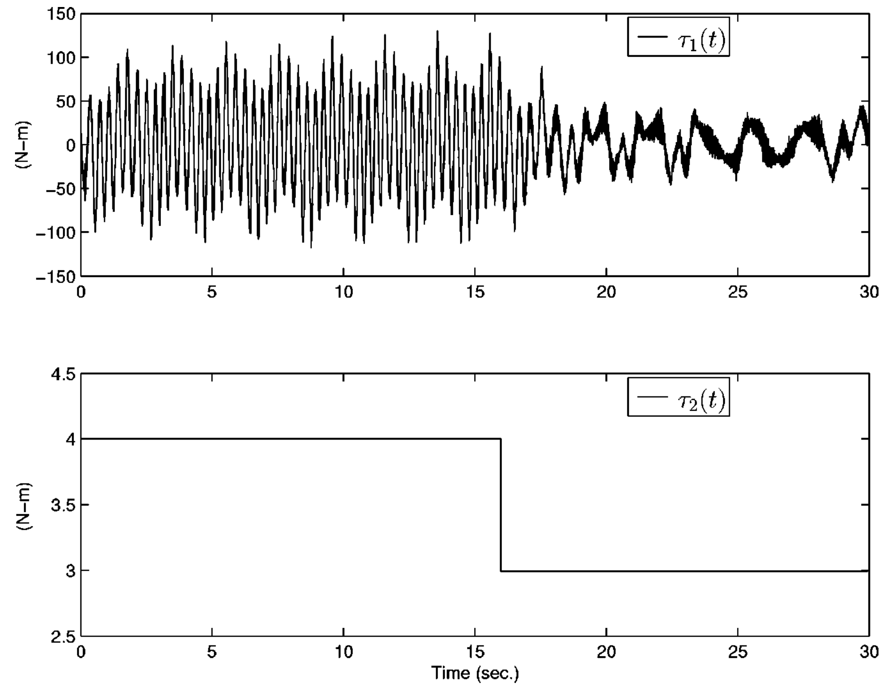

Fig. 5 Motor control torques of base link [$\tau_1(t)$, top plot] and elbow link [$\tau_2(t)$, bottom plot] are shown

$$\ddot{e}_1+2\xi\omega_n\dot{e}_1+\omega_n^2e_1$$
$$=\frac{1}{I}(\tau_1-\dot{I}\dot{q}_1+d-f_1-I\ddot{q}_{1d}+2I\xi\omega_n\dot{e}_1+I\omega_n^2e_1)$$
$$=\frac{1}{I}(\tau_1-\dot{I}\dot{q}_1+\overline{d}-I\ddot{q}_{1d}+2I\xi\omega_n\dot{e}_1+I\omega_n^2e_1)$$

$$+\frac{1}{I}\left[\left(1+\frac{p_3}{p_2}c_2\right)f_2-f_1\right] \tag{50}$$

where ξ, ω_n are two positive constants, and $\overline{d}$ is given by

$$\overline{d}=p_3\left[(\dot{q}_1+\dot{q}_2)^2+\frac{p_3}{p_2}c_2\dot{q}_1^2+\frac{2p_3}{p_2}c_2\dot{q}_1\dot{q}_2\right]s_2-\left(1+\frac{p_3}{p_2}c_2\right)\tau_2$$

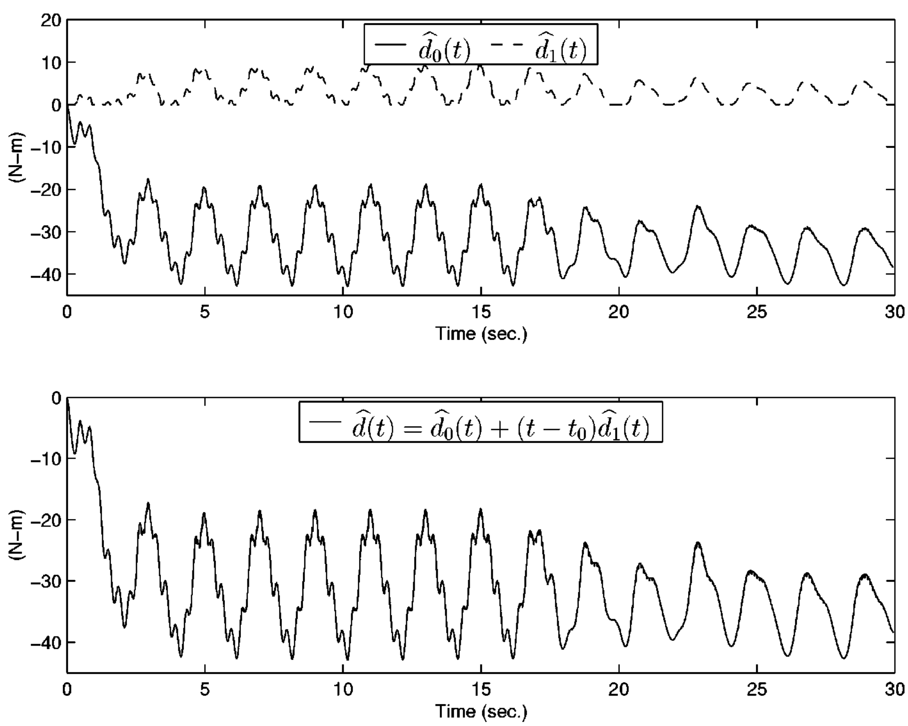

Fig. 6 Estimated disturbance parameters $\hat{d}_0(t)$ and $\hat{d}_1(t)$ are shown in the top plot. The estimate of the disturbance $\hat{d}(t)=\hat{d}_0(t)+(t-t_0)\hat{d}_1(t)$ is shown in the bottom plot.

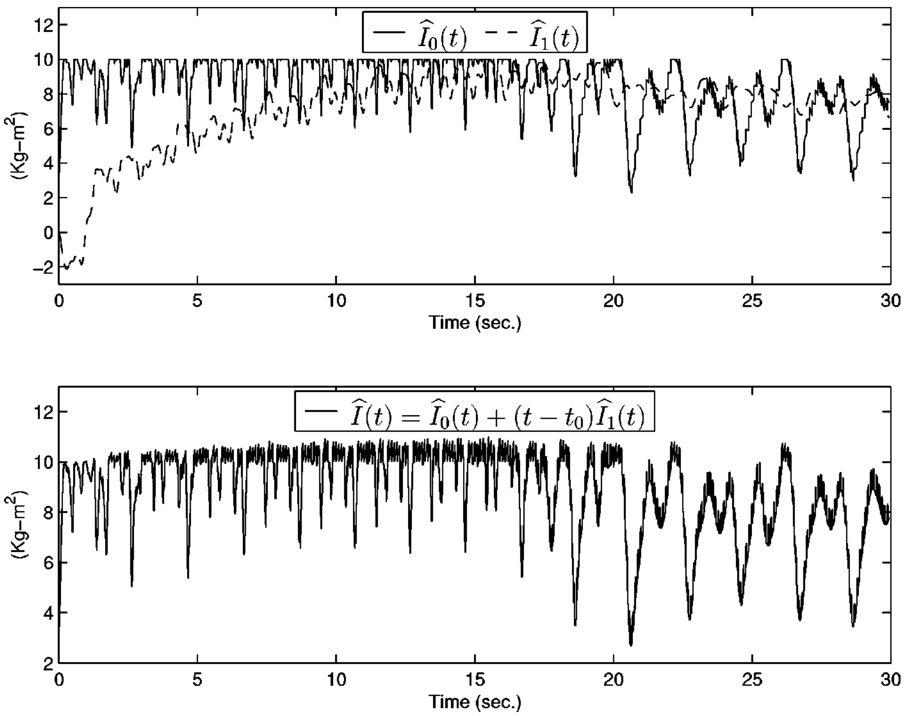

Fig. 7 Estimated inertia parameters $\hat{I}_0(t)$ and $\hat{I}_1(t)$ are shown in the top plot. The estimate of the inertia $\hat{I}(t)=\hat{I}_0(t)+(t-t_0)\hat{I}_1(t)$ is shown in the bottom plot.

Now assuming that the true values of all the constant and time-varying parameters are known, an ideal nonadaptive controller is given by

$$\tau_1=\dot{I}\dot{q}_1-\overline{d}+I\ddot{q}_{1d}-2I\xi\omega_n\dot{e}_1-I\omega_n^2e_1+\delta_{\tau_1} \tag{51}$$

where δ_{τ_1} is a robustness term to account for the unknown terms involving friction. Notice that the term $\overline{d}$ in the control law can be computed based on the measurements and constant parameters p_1, p_2 and p_3. Substitution of the control law (51), into (50) results in

$$\ddot{e}_1+2\xi\omega_n\dot{e}_1+\omega_n^2e_1=\frac{1}{I}\left[\delta_{\tau_1}+\left(1+\frac{p_3}{p_2}c_2\right)f_2-f_1\right] \tag{52}$$

In the following, the robustness term δ_{τ_1} will be designed based on bounds on f_1 and f_2. Consider the viscous plus Coulomb friction models for f_1 and f_2. Then f_1 and f_2 can be bounded as given below:

$$|f_1|\leq F_{v_1}|\dot{q}_1|+F_{c_1}$$

$$|f_2|\leq F_{v_2}|\dot{q}_2|+F_{c_2}$$

where F_{v_1}, F_{c_1}, F_{v_2}, and F_{c_1} are bounds on the viscous and Coulomb friction coefficients. Therefore, the uncertain term in the right-hand side of (52) can be bounded as given below:

$$\left|\left(1+\frac{p_3}{p_2}c_2\right)f_2-f_1\right|\leq\mu\triangleq\left(1+\frac{p_3}{p_2}\right)(F_{v_2}|\dot{q}_2|+F_{c_2})+F_{v_1}|\dot{q}_1| + F_{c_1}$$

Now, the robustness term in the controller can be chosen as

$$\delta_{\tau_1}=-\frac{\dot{e}_1}{|\dot{e}_1|+\varepsilon}\mu \tag{53}$$

where $\varepsilon>0$ is a small constant.

The experimental results for the non-adaptive controller are shown in Fig. 9. The parameters used in the experiment are $\xi=1$, $\omega_n=35$, $F_{v_1}=F_{v_2}=0.1$, $\varepsilon=0.05$, $F_{c_1}=8$, and $F_{c_2}=2$. In Fig. 9, the top plot shows the tracking error of the base link, the middle plot shows the control input to the base link, and the bottom plot is the input torque to the elbow link.

Comparing with the experimental results of the proposed adaptive controller, we can observe that the tracking error using the ideal nonadaptive controller is smaller as expected because it assumes full knowledge of both the time-varying parameters and disturbances. But the performance improvement is not significant. Further, we can observe that the control inputs are comparable.

6 Conclusion

A new adaptive controller for mechanical systems with time-varying parameters and disturbances was proposed. Each time-varying parameter/disturbance was expanded as a finite length polynomial of time and a residue using Taylor's formula. The coefficients of the finite length polynomial are assumed to be constant in a small interval of time. Based on the local approximation of the time-varying parameters and/or disturbances, an adaptive controller was developed for trajectory tracking. The unknown coefficients within each time interval were estimated using a gradient projection algorithm. The tracking error was shown to be ultimately bounded within a certain neighborhood of zero; the size of the neighborhood depends on the choice of the control gains.

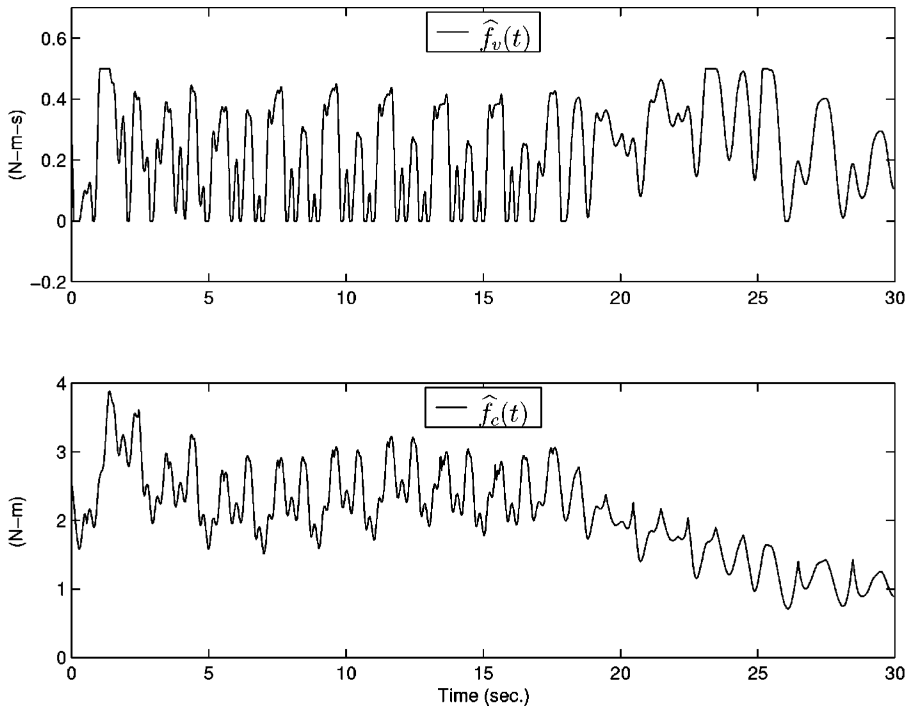

Fig. 8 Estimated friction parameters $\hat{f}_v(t)$ and $\hat{f}_c(t)$ are shown

Using a two-link planar manipulator system, a novel experiment platform was designed to create a time-varying inertia system with time-varying disturbances. This platform was used to validate the proposed adaptive controller experimentally. Further, an ideal nonadaptive controller that assumes full knowledge of the time-varying parameters and disturbances was also implemented. The performance of the proposed adaptive controller was comparable to an ideal nonadaptive controller.

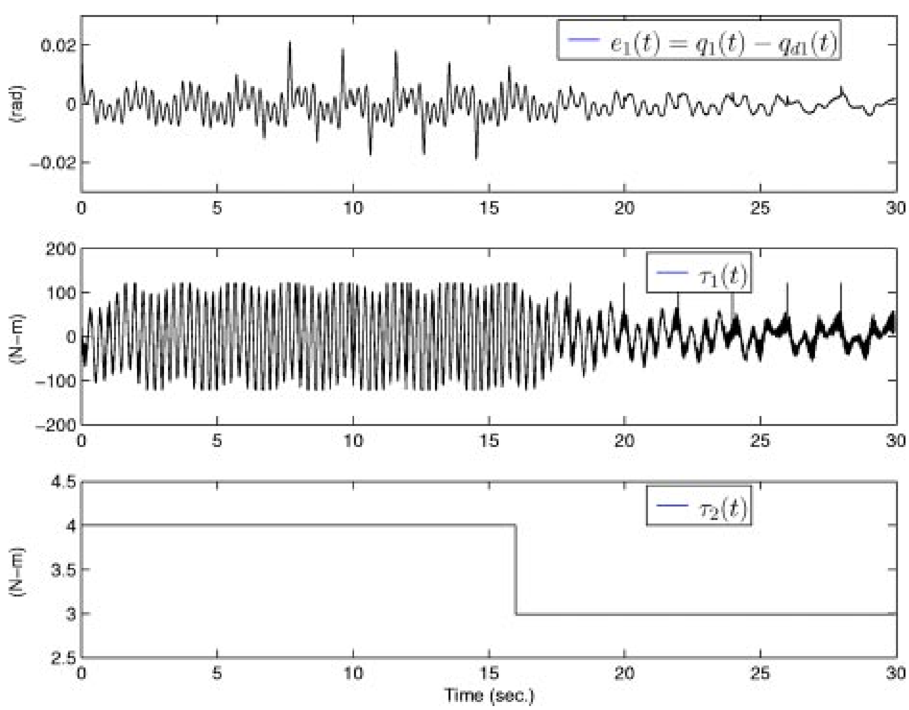

Fig. 9 Experimental results from the ideal nonadaptive robust controller given by (51) and (53). Tracking error of the base link [$e_1(t)$, top plot], motor control torques of the base link [$\tau_1(t)$, middle plot] and the elbow link [$\tau_2(t)$, bottom plot] are shown.

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References

[1] Narendra, K. S., and Annaswamy, A. M., 1989, *Stable Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ.
[2] Song, Y. D., and Middleton, R. H., 1992, ''Dealing With the Time-Varying Parameter Problem of Robot Manipulators Performing Path Tracking Tasks,'' IEEE Trans. Autom. Control, **37**, pp. 1597–1601.
[3] Reed, J. S., and Ioannou, P. A., 1989, ''Instability Analysis and Robust Adaptive Control of Robotic Manipulators,'' IEEE Trans. Rob. Autom., **5**, pp. 381–386.
[4] Yao, B., and Tomizuka, M., 1996, ''Smooth Robust Adaptive Sliding Mode Control of Robot Manipulators With Guaranteed Transient Performance,'' ASME J. Dyn. Syst., Meas., Control, **118**, pp. 764–775.
[5] Tomei, P., 1999, ''Robust Adaptive Control of Robots With Arbitrary Transient Performance and Disturbance Attenuation,'' IEEE Trans. Autom. Control, **44**, pp. 654–658.
[6] Pagilla, P. R., Yu, B., and Pau, K. L., 2000, ''Adaptive Control of Time-Varying Mechanical Systems: Analysis and Experiments,'' Mechatronics, **5**, pp. 410–418.
[7] Tsakalis, K. S., and Ioannou, P. A., 1993, *Linear Time-Varying Systems: Control and Adaptation*, Prentice-Hall, Englewood Cliffs, NJ.
[8] Xianya, X., and Evans, R. J., 1984, ''Adaptive Control of Discrete-Time Time-Varying Systems With Unknown Deterministic Disturbances,'' IEE Proc.-D: Control Theory Appl., **131**, Pt. D, pp. 81–84.
[9] Zheng, L., 1988, ''Discrete-Time Adaptive Control for Time-Varying Systems Subject to Unknown Fast Time-Varying Deterministic Disturbances,'' IEE Proc.-D: Control Theory Appl., **135**, Pt. D, pp. 445–450.
[10] Joensen, A., Madsen, H., Nielsen, H. A., and Nielsen, T. S., 2000, ''Tracking Time-Varying Parameters With Local Regression,'' Automatica, **36**, pp. 1199–1204.
[11] Fan, J., and Gijbels, I., 1996, *Local Polynomial Modelling and its Applications*, Chapman and Hall, London.
[12] Dieudonné, J., 1960, *Foundations of Modern Analysis*, Academic Press, New York.
[13] Greenwood, D. T., 1977, *Classical Dynamics*, Prentice-Hall, Englewood Cliffs, NJ.
[14] Pomet, J. B., and Praly, L., 1992, ''Adaptive Nonlinear Reguation: Estimation From the Lyapunov Equation,'' IEEE Trans. Autom. Control, **37**, pp. 729–740.
[15] Leitmann, G., 1981, ''On the Efficacy of Nonlinear Control in Uncertain Linear Systems,'' ASME J. Dyn. Syst., Meas., Control, **103**, pp. 95–102.
[16] Ioannou, P. A., and Sun, J., 1996, *Robust Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ.