A Decentralized Output Feedback Controller for a Class of Large-Scale Interconnected Nonlinear Systems

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The decentralized output feedback control problem for a class of large-scale interconnected nonlinear systems is considered. The nonlinear interconnection function of each subsystem is assumed to satisfy a quadratic constraint on the entire state of the large-scale system. A decentralized estimated state feedback controller and a decentralized observer are designed for each subsystem. Sufficient conditions, for each subsystem, under which the proposed controller and observer can achieve exponential stabilization of the overall large-scale system are developed. Simulation results on a numerical example are given to verify the proposed design. [DOI: 10.1115/1.1870047]

1 Introduction

Large-scale interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing processes, transportation, and communication. An important motivation for the design of decentralized schemes is that the information exchange between subsystems of a large-scale system is not needed; thus, the individual subsystem controllers are simple and use only locally available information. Decentralized control of large-scale systems has received considerable interest in the systems and control literature. A large body of literature in decentralized control of large-scale systems can be found in [1]. In [2], a survey of early results in decentralized control of large-scale systems was given. Decentralized control schemes that can achieve desired robust performance in the presence of uncertain interconnections can be found in [3–5]. A decentralized control scheme for robust stabilization of a class of nonlinear systems using the linear matrix inequalities (LMI) framework was proposed in [6].

In many practical situations, complete state measurements are not available at each individual subsystem for decentralized control; consequently, one has to consider decentralized feedback control based on measurements only or design decentralized observers to estimate the state of individual subsystems that can be used for estimated state feedback control. There has been a strong research effort in literature towards development of decentralized control schemes based on output feedback via construction of decentralized observers. Early work in this area can be found in [1,3,7]. Subsequent work in [8–12] has focused on the decentralized output feedback problem for a number of special class of large-scale nonlinear systems. The design of an observer-based output feedback controller is a challenging problem for nonlinear systems; it is well known that the separation principle may not be applicable to nonlinear systems [13]. In [14], the decentralized controller and observer design problems were formulated in the LMI framework for large-scale systems with nonlinear interconnections that are quadratically bounded. Autonomous linear decentralized observer-based output feedback controllers for all subsystems were obtained. The existence of a stabilizing controller and observer depended on the feasibility of solving an optimization problem in the LMI framework; further, for a solution to exist, this formulation also required, for each subsystem, that the number of control inputs must be equal to the dimension of the state.

In this paper, we consider a class of large-scale systems with quadratically bounded nonlinear interconnections as in [14]. We design a decentralized controller and observer that can achieve global exponential stabilization under two sufficient conditions. The tools used in the paper are related to the concept of distance to uncontrollability (unobservability) of a pair of matrices $(A,B)((C,A))$ [15–17]. As opposed to the LMI framework, our design does not require as many control inputs as the number of state variables for each subsystem; further, the proposed design results in computable sufficient conditions for each subsystem as opposed to solving an optimization problem for the overall large-scale system. Insights into the problem are provided by considering various special cases which are practically relevant.

The rest of the paper is organized as follows. In Sec. 2, the class of large-scale systems is given with a discussion of the problem and related results available in literature. Some existing results that will be used in the developments of the paper are given in Sec. 3. In Sec. 4, the proposed decentralized controller/observer structure is given; sufficient conditions under which exponential stabilization is achieved are also derived. Simulation results on an example are given in Sec. 5. Section 6 summarizes the paper and highlights some future research topics on the problem.

2 Problem Formulation

The following notation is used. The set of real numbers is denoted by $\mathbb{R}$. The terms $\lambda_{\min}(M), \lambda_{\max}(M), M^H$, and M^T denote the minimum eigenvalue, the maximum eigenvalue, the complex conjugate transpose, and the transpose of the matrix M, respectively. $M>0(\geq 0)$ denotes that the matrix M is symmetric positive definite (symmetric positive semidefinite). $\sigma_{\min}(M) \triangleq \sqrt{\lambda_{\min}(M^H M)}$. The spectral norm of the matrix M is denoted by $\|M\|$. The identity matrix is denoted by I. The term diag $(M_1, \ldots, M_n)$ denotes a block diagonal matrix with M_1 to M_n as its diagonal blocks.

The following class of large-scale interconnected nonlinear systems is considered:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + h_i(t,x), \quad x_i(t_0) = x_{i0} \tag{1a}$$

$$y_i(t) = C_i x_i(t) \tag{1b}$$

where $x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, y_i \in \mathbb{R}^{\ell_i}, h_i \in \mathbb{R}^{n_i}, t_0$, and x_{i0} are the state, input, output, nonlinear interconnection function, initial time, and initial state of the ith subsystem. System (1) consists of N subsystems, that is, $i=1:N$. The interconnections are assumed to be piecewise-continuous functions in both variables, and satisfy the quadratic constraints [14]

$$h_i^T(t,x) h_i(t,x) \leq \alpha_i^2 x^\mathsf{T} H_i^\mathsf{T} H_i x \tag{2}$$

where $\alpha_i > 0$ are interconnection bounds, H_i are bounding matrices, and $x^\mathsf{T} = [x_1^\mathsf{T}, x_2^\mathsf{T}, \ldots, x_N^\mathsf{T}]$ is the state of the overall system. It is assumed that (i) α_i and $\|H_i\|$ are known, (ii) (A_i, B_i) is controllable, and (iii) (C_i, A_i) is observable. Without loss of generality, it is assumed that A_i is Hurwitz (Remark 2 in Sec. 4 discusses this aspect). One specific practical application whose system model conforms to (1) with the quadratic interconnection bounds (2) is a multimachine power system consisting of N interconnected ma-

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chines with steam valve control; the dynamic model is discussed in [18].

The overall system (1) can be rewritten as

$$\dot{x}(t) = A_D x(t) + B_D u(t) + h(t,x), \quad x(t_0) = x_0 \tag{3a}$$

$$y(t) = C_D x(t) \tag{3b}$$

where $A_D = \mathrm{diag}(A_1,\dots,A_N)$, $B_D = \mathrm{diag}(B_1,\dots,B_N)$, $C_D = \mathrm{diag}(C_1,\dots,C_N)$, $u^\top = [u_1^\top,\dots,u_N^\top]$, $y^\top = [y_1^\top,\dots,y_N^\top]$, and $h^\top = [h_1^\top,\dots,h_N^\top]$. The nonlinear interconnections $h(t,x)$ are bounded as follows:

$$h^\top(t,x)h(t,x) \le x^\top \Gamma x \tag{4}$$

where $\Gamma = \Sigma_{i=1}^{N} \alpha_i^2 H_i^\top H_i$. The pair (A_D, B_D) is controllable and the pair (C_D, A_D) is observable.

Since the system (3) is linear with nonlinear interconnections, a common question to ask is under what conditions can we design a decentralized linear controller and a decentralized linear observer that will stabilize the system in the presence of bounded nonlinear interconnections. Towards solving this problem, one can consider the following linear decentralized controller and observer:

$$u(t) = K_D \hat{x}(t) \tag{5}$$

$$\dot{\hat{x}}(t) = A_D \hat{x}(t) + B_D u(t) + L_D(y(t) - C_D \hat{x}(t)) \tag{6}$$

where $K_D = \mathrm{diag}(K_1,\dots,K_N)$ and $L_D = \mathrm{diag}(L_1,\dots,L_N)$ are the controller and observer gain matrices, respectively. Rewriting (3) and (6) in the coordinates $x(t)$ and $\tilde{x}(t)$, where $\tilde{x}(t) \triangleq x(t) - \hat{x}(t)$ is the estimation error, the closed-loop dynamics is

$$\begin{aligned}\dot{x}(t) &= (A_D + B_D K_D)x(t) - B_D K_D \tilde{x}(t) + h(t,x)\\ \dot{\tilde{x}}(t) &= (A_D - L_D C_D)\tilde{x}(t) + h(t,x)\end{aligned} \tag{7}$$

Two broad methods are used to design observer-based decentralized output feedback controllers for large-scale systems: (1) Design local observer and controller for each subsystem independently, and check the stability of the overall closed-loop system. In this method, the interconnection in each subsystem is regarded as an unknown input [17,10]. (2) Design the observer and controller by posing the output feedback stabilization problem as an optimization problem. The optimization approach using LMIs can be found in [14]. We give a brief overview of this approach. It is assumed that H_i is known. The controller gain K_D and the observer gain L_D are obtained from the following minimization problem, provided it is feasible.

Minimize $\Sigma_{i=1}^{N} \gamma_i$ subject to

$$\tilde{P}_1 > 0, \quad \tilde{P}_2 > 0,$$

$$\begin{bmatrix} A_D^\top \tilde{P}_1 + \tilde{P}_1 A_D + M_D^\top + M_D & -M_D & \tilde{P}_1 & H_1^\top & \cdots & H_N^\top \\ -M_D^\top & A_D^\top \tilde{P}_2 + \tilde{P}_2 A_D - C_D^\top N_D^\top - N_D C_D & \tilde{P}_2 & 0 & \cdots & 0 \\ \tilde{P}_1 & \tilde{P}_2 & -I & 0 & \cdots & 0 \\ H_1 & 0 & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N & 0 & 0 & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \tag{8}$$

where $\gamma_i = 1/\alpha_i^2$, and $\tilde{P}_1 B_D K_D = M_D$, $\tilde{P}_2 L_D = N_D$. Under the assumption that the optimization problem, (8), is feasible, the gain matrices K_D and L_D were extracted as follows: $K_D = B_D^{-1}\tilde{P}_1^{-1} M_D$ and $L_D = \tilde{P}_2^{-1} N_D$. The LMI formulation given above requires the invertibility of the input matrix B_D; that is, it requires as many independent control inputs as the number of state variables in each subsystem. Although the formulation as an optimization problem using the LMI framework is quite elegant from a numerical perspective, as it not only computes the gains but also maximizes the interconnection bounds, it can be applied only to a restrictive class of systems in which the input matrix is invertible. Further, obtaining block diagonal positive definite solutions, $\tilde{P}_1$ and $\tilde{P}_2$, from the optimization problem (8) in itself is a challenging problem; the solutions need to be block diagonal for the computed K_D and L_D to be block diagonal; otherwise, one does not get a decentralized solution.

Notice that, because of the nature of the interconnection, $h_i(t,x)$, in some cases, system (1) may not be stabilizable even with full-state feedback control. For example,

$$\dot{x}_1 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_1} x_1 + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_1} u_1 + \gamma_1 \begin{bmatrix} x_{11} - x_{12} \\ x_{21} \end{bmatrix}, \quad y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1 \tag{9}$$

If $\gamma_1 = 1$, then the first state of x_1, x_{11}, has the dynamics $\dot{x}_{11} = x_{11}$, which is unstable and we lose controllability of the system. One cannot design a controller to stabilize the system (9) with the given interconnection, although (A_1, B_1) is controllable. From the example, it is clear that the structure and bounds of the interconnections will affect controllability of subsystems. The same holds true for observability of the system.

The objective of the paper is to design a totally decentralized observer-based linear controller that robustly regulates the state of the overall system without any information exchange between subsystems. We reduce the problem of decentralized exponential stabilization of the large-scale system via output feedback into the existence of symmetric positive definite solutions of two algebraic Ricatti equations (AREs). Further, we develop sufficient conditions for the existence of symmetric positive definite solutions.

3 Preliminaries

Definition 1: The real number $\delta(M,N)$ is defined as

$$\delta(M,N) \triangleq \min_{\omega \in \mathbb{R}} \sigma_{\min} \begin{bmatrix} i\omega I - M \\ N \end{bmatrix} \tag{10}$$

where $i = \sqrt{-1}$, $M \in \mathbb{R}^{n\times n}$, $N \in \mathbb{R}^{p\times n}$. The distance between a pair (A,C) and the set of pairs with an unobservable purely imaginary mode is given by $\delta(A,C)$. Similarly, $\delta(A^\top, B^\top)$ gives the distance between the pair (A,B) and the set of pairs with an uncontrollable purely imaginary mode. See [19] for a discussion of the number δ and a bisection algorithm for computing it.

Lemma 1: [19] Consider the algebraic Ricatti equation

$$A^{\mathsf{T}}P + PA + PRP + Q = 0 \tag{11}$$

If $R=R^{\mathsf{T}} \geqslant 0, Q=Q^{\mathsf{T}} > 0$, A is Hurwitz, and the associated Hamiltonian matrix $\mathcal{H}=\left[\begin{smallmatrix} A & R \\ -Q & -A^{\mathsf{T}} \end{smallmatrix}\right]$ is hyperbolic, i.e., $\mathcal{H}$ has no eigenvalues on the imaginary axis, then there exists a unique $P=P^{\mathsf{T}}>0$, which is the solution of the ARE (11).

Lemma 2: [19,20] Let $\gamma \geqslant 0$ and define $\mathcal{H}_\gamma=\left[\begin{smallmatrix} A & I \\ C^{\mathsf{T}}C-\gamma^2 I & -A^{\mathsf{T}} \end{smallmatrix}\right]$. $\mathcal{H}_\gamma$ is hyperbolic if and only if $\gamma < \delta(A,C)$.

Lemma 3: [21] Assume $A, Q_2, R \in \mathbb{R}^{n\times n}, Q_2=Q_2^{\mathsf{T}}$ and $R=R^{\mathsf{T}} > 0$. If $P_2=P_2^{\mathsf{T}}>0$ satisfies

$$A^{\mathsf{T}}P_2 + P_2A + P_2RP_2 + Q_2 = 0$$

and $Q_1=Q_1^{\mathsf{T}}$ such that $Q_1 \leqslant Q_2$, then there exists a $P_1=P_1^{\mathsf{T}}>0$ such that $P_1 \geqslant P_2$, and

$$A^{\mathsf{T}}P_1 + P_1A + P_1RP_1 + Q_1 = 0.$$

4 The Decentralized Output Feedback Controller Design

Consider the following linear decentralized controller and observer for the ith subsystem:

$$u_i(t) = K_i\hat{x}_i(t) \tag{12}$$

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iu_i(t) + L_i(y_i(t) - C_i\hat{x}_i(t)) \tag{13}$$

where K_i and L_i are the controller and observer gain matrices. Substituting these into the system, (1), we obtain

$$\dot{x}_i(t) = (A_i + B_iK_i)x_i(t) - B_iK_i\tilde{x}_i(t) + h_i(t,x) \tag{14}$$

$$\dot{\tilde{x}}_i(t) = (A_i - L_iC_i)\tilde{x}_i(t) + h_i(t,x) \tag{15}$$

where $\tilde{x}_i=x_i-\hat{x}_i$. For simplicity define the following: $A_{Bi}=B_iK_i$ and $A_{Ci}=L_iC_i$. Consider the following Lyapunov function candidate:

$$V(x,\tilde{x}) = \sum_{i=1}^{N}\left(x_i^{\mathsf{T}}P_ix_i + \tilde{x}_i^{\mathsf{T}}\tilde{P}_i\tilde{x}_i\right). \tag{16}$$

The time derivative of $V(x,\tilde{x})$ along the trajectories of (14) and (15) is given by

$$\begin{aligned}\dot{V}(x,\tilde{x}) = \sum_{i=1}^{N}\Big\{&x_i^{\mathsf{T}}[(A_i+A_{Bi})^{\mathsf{T}}P_i + P_i(A_i+A_{Bi})]x_i + \tilde{x}_i^{\mathsf{T}}[(A_i-A_{Ci})^{\mathsf{T}}\tilde{P}_i \\ &+ \tilde{P}_i(A_i-A_{Ci})]\tilde{x}_i \underbrace{- \tilde{x}_i^{\mathsf{T}}A_{Bi}^{\mathsf{T}}P_ix_i - x_i^{\mathsf{T}}P_iA_{Bi}\tilde{x}_i} + \underbrace{h_i^{\mathsf{T}}P_ix_i + x_i^{\mathsf{T}}P_ih_i} \\ &\underbrace{+ h_i^{\mathsf{T}}\tilde{P}_i\tilde{x}_i + \tilde{x}_i^{\mathsf{T}}\tilde{P}_ih_i}\Big\}\end{aligned} \tag{17}$$

Using the inequality

$$X^{\mathsf{T}}Y + Y^{\mathsf{T}}X \leqslant X^{\mathsf{T}}X + Y^{\mathsf{T}}Y, \quad X,Y \in \mathbb{R}^{m\times n} \tag{18}$$

for terms with under braces in (17), we obtain

$$\tilde{x}_i^{\mathsf{T}}(-A_{Bi})^{\mathsf{T}}P_ix_i + x_i^{\mathsf{T}}P_i(-A_{Bi})\tilde{x}_i \leqslant \tilde{x}_i^{\mathsf{T}}A_{Bi}^{\mathsf{T}}A_{Bi}\tilde{x}_i + x_i^{\mathsf{T}}P_iP_ix_i \tag{19a}$$

$$h_i^{\mathsf{T}}P_ix_i + x_i^{\mathsf{T}}P_ih_i \leqslant h_i^{\mathsf{T}}h_i + x_i^{\mathsf{T}}P_iP_ix_i \tag{19b}$$

$$h_i^{\mathsf{T}}\tilde{P}_i\tilde{x}_i + \tilde{x}_i^{\mathsf{T}}\tilde{P}_ih_i \leqslant h_i^{\mathsf{T}}h_i + \tilde{x}_i^{\mathsf{T}}\tilde{P}_i\tilde{P}_i\tilde{x}_i \tag{19c}$$

Each interconnection function, $h_i(t,x)$, satisfies

$$h_i^{\mathsf{T}}(t,x)h_i(t,x) \leqslant \alpha_i^2x^{\mathsf{T}}H_i^{\mathsf{T}}H_ix \leqslant \alpha_i^2\nu_ix^{\mathsf{T}}x \tag{20}$$

where $\nu_i=\lambda_{\max}(H_i^{\mathsf{T}}H_i)$. We also have

$$\sum_{i=1}^{N}2h_i^{\mathsf{T}}(t,x)h_i(t,x) \leqslant \sum_{i=1}^{N}2\alpha_i^2\nu_i(x_1^{\mathsf{T}}x_1 + \cdots + x_N^{\mathsf{T}}x_N) = \gamma^2\sum_{i=1}^{N}x_i^{\mathsf{T}}x_i \tag{21}$$

where $\gamma^2 \triangleq \Sigma_{i=1}^{N}2\alpha_i^2\nu_i$. Using (19) and (21), in (17) we have

$$\begin{aligned}\dot{V}(x,\tilde{x}) \leqslant \sum_{i=1}^{N}\Big\{&x_i^{\mathsf{T}}\Big[(A_i+A_{Bi})^{\mathsf{T}}P_i + P_i(A_i+A_{Bi}) + 2P_iP_i + \gamma^2I\Big]x_i \\ &+ \tilde{x}_i^{\mathsf{T}}\Big[(A_i-A_{Ci})^{\mathsf{T}}\tilde{P}_i + \tilde{P}_i(A_i-A_{Ci}) + A_{Bi}^{\mathsf{T}}A_{Bi} + \tilde{P}_i\tilde{P}_i\Big]\tilde{x}_i\Big\}\end{aligned} \tag{22}$$

Choose the following gain matrices:

$$K_i = -(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}P_i, \quad L_i = \varepsilon_i\tilde{P}_i^{-1}C_i^{\mathsf{T}}/2, \quad \varepsilon_i > 0 \tag{23}$$

Substituting the gains into (22), we have

$$\begin{aligned}\dot{V}(x,\tilde{x}) \leqslant \sum_{i=1}^{N}\{&x_i^{\mathsf{T}}[A_i^{\mathsf{T}}P_i + P_iA_i + 2P_i(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}})P_i + \gamma^2I]x_i \\ &+ \tilde{x}_i^{\mathsf{T}}[A_i^{\mathsf{T}}\tilde{P}_i + \tilde{P}_iA_i + \tilde{P}_i\tilde{P}_i + \tilde{Q}_{i1} - \varepsilon_iC_i^{\mathsf{T}}C_i]\tilde{x}_i\}\end{aligned} \tag{24}$$

where $\tilde{Q}_{i1} \triangleq K_i^{\mathsf{T}}B_i^{\mathsf{T}}B_iK_i$. From the above, we have the following result. For some $\eta_i>0$ and $\tilde{\eta}_i>0$, if there exist positive definite solutions to the AREs

$$A_i^{\mathsf{T}}P_i + P_iA_i + 2P_i(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}})P_i + \gamma^2I + \eta_iI = 0 \tag{25}$$

$$A_i^{\mathsf{T}}\tilde{P}_i + \tilde{P}_iA_i + \tilde{P}_i\tilde{P}_i + \tilde{Q}_{i1} + \tilde{\eta}_iI - \varepsilon_iC_i^{\mathsf{T}}C_i = 0 \tag{26}$$

then

$$\dot{V}(x,\tilde{x}_i) \leqslant -\sum_{i=1}^{N}[\eta_ix_i^{\mathsf{T}}x_i + \tilde{\eta}_i\tilde{x}_i^{\mathsf{T}}\tilde{x}_i] \tag{27}$$

As a result, the problem reduces to the following: If there are positive definite solutions to the AREs (25) and (26), then $V(x,\tilde{x})$ is a Lyapunov function; that is, $V(x,\tilde{x})$ is positive and $\dot{V}(x,\tilde{x})$ is negative.

Remark 1: The control gain matrix K_i given by (23) requires that $B_i^{\mathsf{T}}B_i$ is invertible; $B_i^{\mathsf{T}}B_i$ is invertible if B_i has full column rank, which is always possible.

Remark 2: If A_i is not stable, then we can stabilize A_i by changing K_i and L_i given by (23), to the following:

$$K_i = -(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}P_i - \overline{K_i}, \quad L_i = \varepsilon_i \tilde{P}_i^{-1}C_i^{\mathsf{T}}/2 + \overline{L_i} \tag{28}$$

where $\overline{K_i}$ and $\overline{L_i}$ are pre-feedback gains such that $A_i^c \triangleq A_i - B_i\overline{K_i}$ and $A_i^o \triangleq A_i - \overline{L_i}C_i$ are Hurwitz. In such a case, A_i in (25) and (26) must be replaced by A_i^c and A_i^o, respectively.

Notice that we cannot design the controller and observer independently, that is, the separation principle does not hold; the ARE (26) depends on the control gain matrix K_i. It should be noted that the above reduction procedure has yielded the following: One can design the controller gain independent of the observer and further, only the first ARE, (25), explicitly depends on the interconnection bounds. The problem now reduces to the following: What are the conditions under which there exist positive definite solutions to the AREs (25) and (26). In the following, two sufficient conditions will be derived.

4.1 Sufficient Conditions. We first consider the ARE (15). The associated Hamiltonian matrix is given by $\mathcal{H}_i = \begin{bmatrix} A_i & R_i \\ -Q_i & -A_i^{\mathsf{T}} \end{bmatrix}$ where $R_i = 2(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}) \geqslant 0, Q_i = (\gamma^2 + \eta_i)I > 0$. The following lemma gives a condition under which H_i is hyperbolic; thus, by lemma 1, it gives a sufficient condition for the existence of a unique $P_i = P_i^{\mathsf{T}} > 0$ to the ARE (25).

Lemma 4: *$\mathcal{H}_i$ is hyperbolic if and only if*

$$\delta\left(A_i^{\mathsf{T}}, \sqrt{2(\gamma^2 + \eta_i)}(B_i^{\mathsf{T}}B_i)^{-1/2}B_i^{\mathsf{T}}\right) > \sqrt{2(\gamma^2 + \eta_i)} \tag{29}$$

Proof: Consider the determinant of the matrix $(sI - \mathcal{H}_i)$

$$\det(sI - \mathcal{H}_i) = \det\begin{bmatrix} sI - A_i & -2\left(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}\right) \\ (\gamma^2 + \eta_i)I & sI + A_i^{\mathsf{T}} \end{bmatrix}$$
$$= (-1)^{n_i}\det\begin{bmatrix} (\gamma^2 + \eta_i)I & sI + A_i^{\mathsf{T}} \\ sI - A_i & -2\left(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}\right) \end{bmatrix}$$

Since $(\gamma^2 + \eta_i)I$ is non-singular, using the formula for determinant of block matrices [[22], p. 650], we obtain

$$\det(sI - \mathcal{H}_i) = (-1)^{n_i} \times \det\underbrace{\left[-2(\gamma^2 + \eta_i)\left(I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}\right) - (sI - A_i)(sI + A_i^{\mathsf{T}})\right]}_{G(s)}$$

From the above equation, s is an eigenvalue of $\mathcal{H}_i$ if and only if $G(s)$ is singular. Hence, to prove that $\mathcal{H}_i$ is hyperbolic, one can prove that $G(-i\omega)$ is nonsingular for all $\omega \in \mathbb{R}$. Notice that

$$G(-i\omega) = -2(\gamma^2 + \eta_i)I - (-i\omega I - A_i)(-i\omega I + A_i^{\mathsf{T}}) + 2(\gamma^2 + \eta_i)B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}}$$
$$= -2(\gamma^2 + \eta_i)I + \begin{bmatrix} i\omega I - A_i^{\mathsf{T}} \\ \sqrt{2(\gamma^2 + \eta_i)}(B_i^{\mathsf{T}}B_i)^{-1/2}B_i^{\mathsf{T}} \end{bmatrix}^H \times \begin{bmatrix} i\omega I - A_i^{\mathsf{T}} \\ \sqrt{2(\gamma^2 + \eta_i)}(B_i^{\mathsf{T}}B_i)^{-1/2}B_i^{\mathsf{T}} \end{bmatrix}$$

Therefore, if (29) is satisfied, then $G(-i\omega) > 0$ for all $\omega \in \mathbb{R}$. Thus, H_i is hyperbolic. This completes the sufficiency part of the proof. The necessary part of the proof is similar to that of lemma 2. ■

The Hamiltonian matrix associated with the ARE (26) is $\tilde{H}_i = \begin{bmatrix} A_i & \tilde{R}_i \\ -\tilde{Q}_i & -A_i^{\mathsf{T}} \end{bmatrix}$ where $\tilde{R}_i = I > 0$ and $\tilde{Q}_i = \tilde{Q}_{i1} + \tilde{\eta}_i I - \varepsilon_i C_i^{\mathsf{T}}C_i$. Choose $\tilde{\eta}_i > 0$ and $\varepsilon_i > 0$ such that $\tilde{Q}_i > 0$. The following lemma 5 gives a condition under which $\mathcal{H}_i$ is hyperbolic; the proof of which is similar to lemma 4. Thus, by lemma 1, it gives a sufficient condition for the existence of a symmetric positive definite solution to the ARE (26).

Lemma 5: *$\mathcal{H}_i$ is hyperbolic if and only if*

$$\sqrt{\lambda_{\max}(\tilde{Q}_{i1}) + \tilde{\eta}_i} < \delta(A_i, C_i) \tag{30}$$

Theorem 1: For the large-scale system given by (3), the decentralized controller and observer as given by (12) and (13) will result in exponential stabilization of the overall system, if (29) and (30) are satisfied for all $i = 1:N$.

Proof: If (29) and (30) are satisfied for all $i = 1:N$, then from lemmas 4, 5, and 1, the AREs (25) and (26) have symmetric positive definite solutions, P_i and $\tilde{P}_i$, respectively. Consequently, one can choose $V(x, \tilde{x})$ given by (16) as the Lyapunov function of the overall system (3). Thus, exponential stabilization of the overall closed-loop system is achieved. ■

4.2 Remarks. *Remark 3*: Since $f(\gamma) \triangleq \sqrt{2}\gamma - \delta(A_i^{\mathsf{T}}, \sqrt{2}\gamma(B_i^{\mathsf{T}}B_i)^{-1/2}B_i^{\mathsf{T}})$ is a continuous function of γ, if $f(\gamma) < 0$, then there exists a $\gamma_1 > \gamma$ such that $f(\gamma_1) < 0$, that is, there exists an $\eta_i > 0$ such that (29) holds. Same arguments hold for condition (30). Hence, instead of checking conditions given by (29) and (30), one can check the following two conditions

$$\sqrt{2}\gamma < \delta\left(A_i^{\mathsf{T}}, \sqrt{2}\gamma(B_i^{\mathsf{T}}B_i)^{-1/2}B_i^{\mathsf{T}}\right), \quad \sqrt{\lambda_{\max}(\tilde{Q}_{i1})} < \delta(A_i, C_i) \tag{31}$$

Notice that the conditions given by (31) guarantee the existence of $\eta_i > 0$ and $\tilde{\eta}_i > 0$, but not their values. Conditions (29) and (30) with specified η_i and $\tilde{\eta}_i$ give the rate of convergence of controller and observer, respectively.

Remark 4: We have the following results when the matrices B_i and C_i are invertible.

Lemma 6: If B_i is invertible, there always exists a symmetric positive definite solution $\tilde{P}_i$ to the ARE (25).

Proof: When B_i is invertible, $I - B_i(B_i^{\mathsf{T}}B_i)^{-1}B_i^{\mathsf{T}} = 0$, as a result, the ARE (25) reduces to Lyapunov equation

$$A_i^{\mathsf{T}}P_i + P_iA_i + (\gamma^2 + \eta_i)I = 0 \tag{32}$$

Since $(\gamma^2 + \eta_i)I > 0$ and A_i is stable, there always exists a $P_i = P_i^{\mathsf{T}} > 0$ satisfying (32) for any $\gamma, \eta_i > 0$. ■

Lemma 7: If C_i is invertible, there always exists a symmetric positive definite matrix $\tilde{P}_i$ to the ARE (26).

Proof: Because $\tilde{Q}_{i1} + \tilde{\eta}_i I$ is a constant matrix, ε_i can be chosen large enough such that

$$\tilde{Q}_i \triangleq \tilde{Q}_{i1} + \tilde{\eta}_i I - \varepsilon_i C_i^{\mathsf{T}}C_i < 0 \tag{33}$$

Notice that $-\tilde{Q}_i = \tilde{G}_i\tilde{G}_i^{\mathsf{T}} > 0$. The ARE (26) reduces to

$$(-A_i)^{\mathsf{T}}\tilde{P}_i + \tilde{P}_i(-A_i) - \tilde{P}_i\tilde{P}_i + \tilde{G}_i\tilde{G}_i^{\mathsf{T}} = 0 \tag{34}$$

Since $(I, -A_i^{\mathsf{T}})$ is observable and $(-A_i^{\mathsf{T}}, \tilde{G}_i)$ is controllable, ARE (34) has a unique positive definite solution $\tilde{P}_i$ [23]. ■

Remark 5: Since the constant ε_i affects the convergence rate of the observation error and the stability of the overall system, a natural question to ask is what happens if we increase–decrease the value of ε_i. The following lemma 8 gives a result related to this; the proof of which follows from lemma 3.

Lemma 8: If the sufficient condition (30) is satisfied for a particular ε_i, then there exists a symmetric positive definite solution to the ARE (25) for any $\varepsilon_i' \geqslant \varepsilon_i$ instead of ε_i. Moreover, the solution corresponding to ε_i' for the ARE (26), $\tilde{P}_i'$, satisfies $\tilde{P}_i' \geqslant \tilde{P}_i$.

Remark 6: The convergence rate of each subsystem observer can be increased by amplifying the observer gain matrix L_i obtained from (23) by $\varepsilon_i'/\varepsilon_i$. Let $L_i = \varepsilon_i'\tilde{P}_i^{-1}C_i^{\mathsf{T}}/2$, where $\tilde{P}_i = \tilde{P}_i^{\mathsf{T}} > 0$ is the solution to the ARE (26) obtained with ε_i. Then the inequality (27) becomes

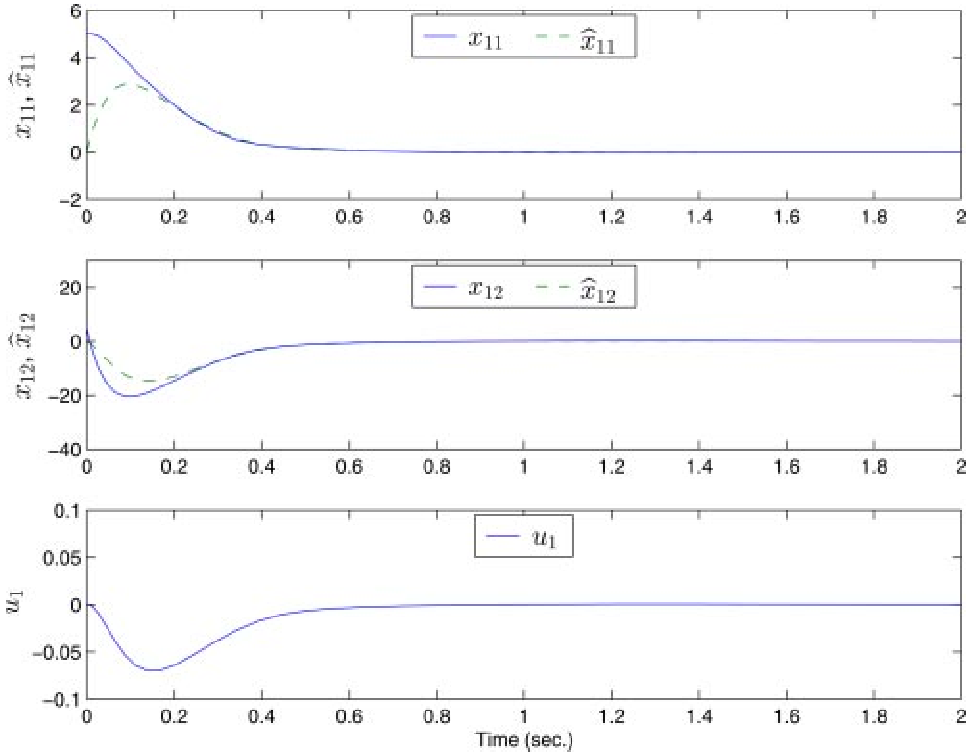

Fig. 1 Subsystem 1 simulation results

$$\dot{V}(x,\tilde{x}) \leqslant -\sum_{i=1}^{N} \left[\eta_i x_i^{\mathsf{T}} x_i + \tilde{\eta}_i \tilde{x}_i^{\mathsf{T}} \tilde{x}_i + (\varepsilon_i' - \varepsilon_i) \tilde{x}_i^{\mathsf{T}} C_i^{\mathsf{T}} C_i \tilde{x}_i \right] \quad (35)$$

Since $\varepsilon_i' - \varepsilon_i > 0$, the convergence rate of $\tilde{x}_i$ to zero is increased.

Remark 7: The inequality (18) used in separating the terms can be quite conservative. Instead of (18), one can use the following inequality

$$X^{\mathsf{T}} Y + Y^{\mathsf{T}} X \leqslant X^{\mathsf{T}} X/\varepsilon + \varepsilon Y^{\mathsf{T}} Y, \quad \varepsilon > 0 \quad (36)$$

The disadvantage of this approach is that one has to choose ε in the design also.

5 Simulation

Consider the following large-scale system:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -125 & -22.5 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + h_1(x), \quad y_1 = [1 \quad 0] x_1,$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -37.5 & -50 & -13.5 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 + h_2(x) x,$$

$$y_2 = [1 \quad 0 \quad 0] x_2$$

where $x_1^{\mathsf{T}} = [x_{11}, x_{12}], x_2^{\mathsf{T}} = [x_{21}, x_{22}, x_{23}], x^{\mathsf{T}} = [x_1^{\mathsf{T}}, x_2^{\mathsf{T}}], h_1(x) = \alpha_1 \cos(x_{22}) H_1 x, h_2(x) = \alpha_1 \cos(x_{11}) H_2 x, \alpha_1 = \alpha_2 = 0.2, H_1 = I_{2,5}/\sqrt{10}$ and $H_2 = I_{3,5}/\sqrt{15}$ are normalized matrices. $I_{i,j}$ denotes an $i \times j$ dimensional matrix with all its elements being 1. The initial conditions were chosen to be $x_1^{\mathsf{T}}(0) = [5,5]$ and $x_2^{\mathsf{T}}(0) = [5,5,5]$. The gain γ is computed based on the values of α_1, α_2, H_1 and H_2 as $\gamma = 0.4$. The following constant gains are chosen: $\varepsilon_1 = 0.5, \varepsilon_2 = 0.125, \eta_1 = 0.1, \tilde{\eta}_1 = 0.5, \eta_2 = 0.01$, and $\tilde{\eta}_2 = 0.2$. It is checked that the conditions given by (29) and (30) are satisfied for both subsystems. The control and observer gain matrices from (23) are $K_1 = [-0.0080, -0.0061], K_2 = [-0.0306, -0.0378, -0.0093], L_1^{\mathsf{T}} = [0.1488, -0.1465], L_2^{\mathsf{T}} = [0.3358, -0.2157, 0.1625]$. To increase the convergence rate of the observers, we choose $100L_1$ and $10L_2$ as the observer gain matrices for the first and second subsystem, respectively, in the simulation.

The simulation results are shown in Figs. 1 and 2. In Fig. 1, the state x_{11} and its estimate $\hat{x}_{11}$, the state x_{12} and its estimate $\hat{x}_{12}$, and the control u_1 are shown in the first, second and third plot, respectively. Figure 2 shows the states x_2, their estimates $\hat{x}_2$, and the control u_2. It can be observed from both figures that the state of the overall system, x, and their estimates, $\hat{x}$, converge to zero.

6 Conclusion

We proposed a decentralized controller and observer for a class of large-scale interconnected nonlinear systems. The interconnecting nonlinearity of each subsystem was assumed to be bounded by a quadratic form of states of the overall system. Local output signals from each subsystem are required to generate the local feedback controller and exact knowledge of the nonlinear interconnection is not required for designing the proposed decentralized controller and observer. Sufficient conditions for the existence of the decentralized controller and observer are given via the analysis of two AREs. Simulation results on a numerical example verify the proposed design.

There are some challenging problems related to the quantity δ. The quantities $\delta(A,C)$ or $\delta(A^{\mathsf{T}}, B^{\mathsf{T}})$ are realization dependent. The properties of δ as a function of various state-space realizations is of importance. In particular, finding the realization of the state-space maximizes the value of δ will be useful.

Acknowledgments

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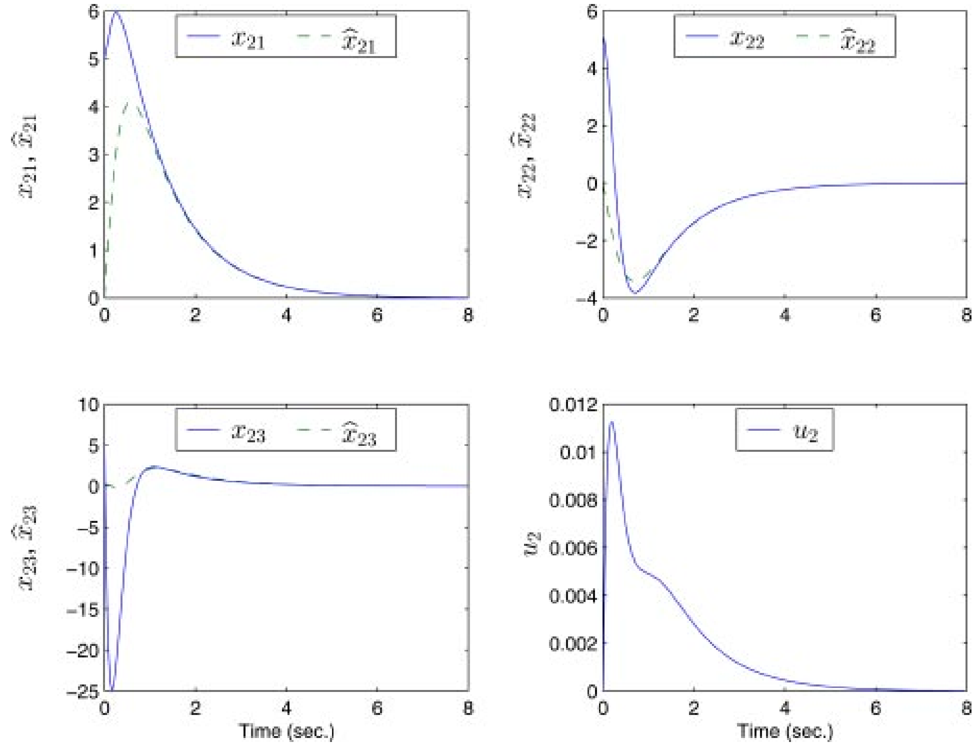

Fig. 2 Subsystem 2 simulation results

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