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Decentralized output feedback control of a class of large-scale interconnected systems

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The focus of the research is on the design of a decentralized output feedback controller for a class of large-scale systems using linear matrix inequalities (LMIs). The class of large-scale systems is characterized by unmatched non-linear interconnection functions that are uncertain but quadratically bounded in the overall system state. An elegant LMI solution to the problem was given in Siljak & Stipanovic (2001, Autonomous decentralized control. *Proceedings of the International Mechanical Engineering Congress and Exposition*. Nashville, TN), but the method requires that the input matrix of each subsystem be invertible, i.e. each subsystem has as many independent control inputs as state variables. We provide an LMI solution that does not require invertibility of the input matrix of each subsystem. Simulation results on an example are given to validate the design.

Keywords: large-scale systems; decentralized control; output feedback control; linear matrix inequalities (LMIs).

1. Introduction

Large-scale interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing processes, transportation and communication. An important motivation for the design of decentralized schemes is that the information exchange between subsystems of a large-scale system is not needed; thus, the individual subsystem controllers are simple and use only locally available information. Decentralized control of large-scale systems has received considerable interest in the systems and control literature. A large body of literature in decentralized control of large-scale systems can be found in Siljak (1991). In Sandell *et al.* (1978), a survey of early results in decentralized control of large-scale systems was given. Decentralized control schemes that can achieve desired robust performance in the presence of uncertain interconnections can be found in Ikeda (1989), Zhang *et al.* (1996) and Gong (1995). Stabilization and tracking using decentralized adaptive controllers were considered in Ioannou (1986) and sufficient conditions were established which guarantee boundedness and exponential convergence of errors; this result is for the case where the relative degree of the transfer function of each decoupled subsystem of the large-scale system is less than or equal to two. Decentralized control schemes that can achieve desired robust performance in the presence of uncertain interconnections can be found in Ikeda (1989). A decentralized control scheme for robust stabilization of a class of non-linear systems using the linear matrix inequalities (LMIs) framework was proposed in Siljak & Stipanovic (2000).

In many practical situations, complete state measurements are not available at each individual subsystem for decentralized control; consequently, one has to consider decentralized feedback control based

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on measurements only or design decentralized observers to estimate the state of individual subsystems that can be used for estimated state feedback control. There has been a strong research effort in literature towards development of decentralized control schemes based on output feedback via construction of decentralized observers. Early work in this area can be found in Viswanadham & Ramakrishna (1982), Ikeda (1989) and Siljak (1991). Two broad methods are used to design observer-based decentralized output feedback controllers for large-scale systems: (1) Design local observer and controller for each subsystem independently and check the stability of the overall closed-loop system. In this method, the interconnection in each subsystem is regarded as an unknown input (Viswanadham & Ramakrishna, 1982; Aldeen & Marsh, 1999). (2) Design the observer and controller by posing the output feedback stabilization problem as an optimization problem. The optimization approach using LMIs can be found in Siljak & Stipanovic (2001).

Recent work in Abdel-Jabbar *et al.* (1998), Aldeen & Marsh (1999), Jiang (2000), Narendra & Oleng (2002), Siljak *et al.* (2002) and Mirkin & Gutman (2003) has focused on the decentralized output feedback problem for a number of special classes of non-linear systems. A partially decentralized state observer was proposed in Abdel-Jabbar *et al.* (1998); an efficient implementation of the decentralized observers was also given. In Aldeen & Marsh (1999), construction of a stable, decentralized observer-based control scheme with unknown inputs was described. A stable adaptive tracking controller using output feedback for a class of non-linear systems was proposed in Jiang (2000). In Narendra & Oleng (2002), considering systems with matched interconnections, it is shown that in strictly decentralized adaptive control systems, it is theoretically possible to asymptotically track the desired output with zero error. A control scheme for robust decentralized stabilization of multimachine power systems, based on LMIs, was proposed in Siljak *et al.* (2002). A decentralized output feedback model reference adaptive controller for systems with delay can be found in Mirkin & Gutman (2003).

In Siljak & Stipanovic (2001), the decentralized controller and observer design problems were formulated in the LMI framework for large-scale systems with non-linear interconnections that are quadratically bounded. Autonomous linear decentralized observer-based output feedback controllers for all subsystems were obtained. The existence of a stabilizing controller and observer depends on the feasibility of solving an optimization problem in the LMI framework. The optimization problem is posed in a fashion that will result in selection of controller and observer gains that will not only stabilize the overall large-scale system but also simultaneously maximize the interconnection bounds. Further, for a solution to exist for the optimization problem, the proposed method also required, for each subsystem, the number of independent control inputs must be equal to the dimension of the state; i.e. invertibility of the each subsystem input matrix was necessary. An output feedback solution based on the concept of distance to controllability/observability was proposed in Pagilla & Zhu (2005); the feasibility of the solution was dependent on satisfying the distance to controllability/observability of pairs of matrices being larger than a certain value.

The contributions of this paper over prior work are summarized in the following. An efficient decentralized output feedback controller for large-scale systems with quadratic interconnections is proposed; the proposed method does not require that for each subsystem the number of state variables be equal to input variables. The proposed LMI solution is obtained as a sequential two-part optimization problem. The feasibility of both parts of the optimization problem is shown and discussed. Further, a variation of the optimization problem is also given which provides restrictions on the size of the controller/observer gains, thus providing constraints on the control effort that can be used.

The remainder of the paper is organized as follows: The decentralized output feedback problem, the solution of Siljak & Stipanovic (2001) and the proposed LMI solution are described in Section 2. Section 3 gives simulation results for an example. Conclusions are given in Section 4.

2. Decentralized output feedback controller design

The following class of large-scale interconnected non-linear systems is considered:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + h_i(t, x), \tag{1a}$$

$$y_i(t) = C_i x_i(t), \tag{1b}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^{l_i}$ and $h_i \in \mathbb{R}^{n_i}$ are the state, input, output and non-linear interconnection function of the ith subsystem and $x = [\, x_1^\top \; x_2^\top \; \dots \; x_N^\top \,]^\top$ is the state of the overall system. The term $h_i(t, x)$ reflects the interconnection of the ith subsystem with other subsystems and the uncertainty dynamics from the ith subsystem itself. The exact expression of $h_i(t, x)$ is unknown but it is assumed to satisfy the quadratic constraints (Siljak & Stipanovic, 2001)

$$h_i^T(t, x) h_i(t, x) \leqslant \alpha_i^2 x^\top H_i^\top H_i x, \tag{2}$$

where $\alpha_i > 0$ are interconnection bounds and H_i are bounding matrices. It is also assumed that (A_i, B_i) is a controllable pair and (C_i, A_i) is an observable pair for all $i \in \mathscr{I} = \{1, 2, \dots, N\}$.

The objective is to design a totally decentralized observer-based linear controller that robustly regulates the state of the overall system without any information exchange between subsystems, i.e. the local controller u_i is constrained to use only local output signal y_i. One specific practical application whose system model conforms to (1) with the quadratic interconnection bounds (2) is a multi-machine power system consisting of N interconnected machines with steam valve control; the dynamic model of this system is discussed in Siljak *et al.* (2002).

The overall system (1) can be rewritten as

$$\dot{x}(t) = A_D x(t) + B_D u(t) + h(t, x), \tag{3a}$$

$$y(t) = C_D x(t), \tag{3b}$$

where $A_D = \text{diag}(A_1, \dots, A_N)$, $B_D = \text{diag}(B_1, \dots, B_N)$, $C_D = \text{diag}(C_1, \dots, C_N)$, $u = [u_1^\top \dots u_N^\top]^\top$, $y = [y_1^\top \dots y_N^\top]^\top$ and $h = [h_1^\top \dots h_N^\top]^\top$. The non-linear interconnections $h(t, x)$ are bounded as follows:

$$h^\top(t, x) h(t, x) \leqslant x^\top \left(\sum_{i=1}^{N} \alpha_i^2 H_i^\top H_i \right) x =: x^\top \Gamma^\top \Gamma x. \tag{4}$$

The pair (A_D, B_D) is controllable and the pair (C_D, A_D) is observable, which is the direct result of each subsystem being controllable and observable.

Since system (3) is linear with non-linear interconnections, a common question to ask is under what conditions can we design a decentralized linear controller and a decentralized linear observer that will stabilize the system in the presence of bounded quadratic interconnections. Towards solving this problem, one can consider the following linear decentralized controller and observer:

$$u(t) = K_D \widehat{x}(t), \tag{5}$$

$$\dot{\widehat{x}}(t) = A_D \widehat{x}(t) + B_D u(t) + L_D(y(t) - C_D \widehat{x}(t)), \tag{6}$$

where $K_D = \text{diag}(K_1, \dots, K_N)$ and $L_D = \text{diag}(L_1, \dots, L_N)$ are the controller and observer gain matrices, respectively. Rewriting (3) and (6) in the coordinates $x(t)$ and $\widetilde{x}(t)$, where $\widetilde{x}(t) \triangleq x(t) - \widehat{x}(t)$

is the estimation error, the closed-loop dynamics is

$$\dot{x}(t) = (A_D + B_D K_D)x(t) - B_D K_D \widetilde{x}(t) + h(t, x), \tag{7a}$$

$$\dot{\widetilde{x}}(t) = (A_D - L_D C_D)\widetilde{x}(t) + h(t, x). \tag{7b}$$

Let

$$A_{\mathrm{c}} \triangleq A_D + B_D K_D, \quad A_{\mathrm{o}} \triangleq A_D - L_D C_D. \tag{8}$$

Consider the following Lyapunov function candidate

$$V(x, \widetilde{x}) = x^\top \bar{P}_c x + \widetilde{x}^\top \bar{P}_o \widetilde{x}. \tag{9}$$

where $\bar{P}_c > 0$ and $\bar{P}_{\mathrm{o}} > 0$.
The time derivative of $V(x, \widetilde{x})$ along the trajectories of (7) is given by

$$\dot{V}(x, \widetilde{x}) = \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}^\top \begin{bmatrix} A_{\mathrm{c}}^\top \bar{P}_{\mathrm{c}} + \bar{P}_{\mathrm{c}} A_{\mathrm{c}} & -\bar{P}_{\mathrm{c}} B_D K_D & \bar{P}_{\mathrm{c}} \\ -K_D^\top B_D^\top \bar{P}_{\mathrm{c}} & A_{\mathrm{o}}^\top \bar{P}_{\mathrm{o}} + \bar{P}_{\mathrm{o}} A_{\mathrm{o}} & \bar{P}_{\mathrm{o}} \\ \bar{P}_{\mathrm{c}} & \bar{P}_{\mathrm{o}} & 0 \end{bmatrix} \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}. \tag{10}$$

The interconnection condition (4) is equivalent to

$$\begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix}^\top \begin{bmatrix} -\Gamma^\top \Gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x \\ \widetilde{x} \\ h \end{bmatrix} \leqslant 0. \tag{11}$$

The stabilization of system (7) requires that

$$\dot{V}(x, \widetilde{x}) < 0 \tag{12}$$

for all $x, \widetilde{x} \neq 0$; together with the condition given by (11), one can obtain (Boyd *et al.*, 1994) that if

$$\begin{bmatrix} A_{\mathrm{c}}^\top \bar{P}_{\mathrm{c}} + \bar{P}_{\mathrm{c}} A_{\mathrm{c}} & -\bar{P}_{\mathrm{c}} B_D K_D & \bar{P}_{\mathrm{c}} \\ -K_D^\top B_D^\top \bar{P}_{\mathrm{c}} & A_{\mathrm{o}}^\top \bar{P}_{\mathrm{o}} + \bar{P}_{\mathrm{o}} A_{\mathrm{o}} & \bar{P}_{\mathrm{o}} \\ \bar{P}_{\mathrm{c}} & \bar{P}_{\mathrm{o}} & 0 \end{bmatrix} - \tau \begin{bmatrix} -\Gamma^\top \Gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} < 0, \tag{13a}$$

$$\bar{P}_{\mathrm{c}} > 0, \quad \bar{P}_{\mathrm{o}} > 0, \quad \tau > 0, \tag{13b}$$

then the inequality (12) is satisfied. Let

$$P_{\mathrm{c}} \triangleq \frac{\bar{P}_{\mathrm{c}}}{\tau}, \quad P_{\mathrm{o}} \triangleq \frac{\bar{P}_{\mathrm{o}}}{\tau}.$$

The condition given by (13) is equivalent to

$$\begin{bmatrix} A_{\mathrm{c}}^\top P_{\mathrm{c}} + P_{\mathrm{c}} A_{\mathrm{c}} + \Gamma^\top \Gamma & -P_{\mathrm{c}} B_D K_D & P_{\mathrm{c}} \\ -K_D^\top B_D^\top P_{\mathrm{c}} & A_{\mathrm{o}}^\top P_{\mathrm{o}} + P_{\mathrm{o}} A_{\mathrm{o}} & P_{\mathrm{o}} \\ P_{\mathrm{c}} & P_{\mathrm{o}} & -I \end{bmatrix} < 0, \tag{14a}$$

$$P_{\mathrm{c}} > 0, \quad P_{\mathrm{o}} > 0. \tag{14b}$$

Considering (4) and (8), and applying the Schur complement to the inequality (14), results in

$$P_c > 0, \quad P_o > 0, \tag{15a}$$

$$\begin{bmatrix} W_C & -P_c B_D K_D & P_c & \alpha_1 H_1^\top & \dots & \alpha_N H_N^\top \\ -(P_c B_D K_D)^\top & W_O & P_o & 0 & \dots & 0 \\ P_c & P_o & -I & 0 & \dots & 0 \\ \alpha_1 H_1 & 0 & 0 & -I & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & \vdots \\ \alpha_N H_N & 0 & 0 & 0 & \dots & -I \end{bmatrix} < 0, \tag{15b}$$

where

$$W_C \stackrel{\triangle}{=} A_D^\top P_c + P_c A_D + (P_c B_D K_D)^\top + (P_c B_D K_D), \tag{16}$$

$$W_O \stackrel{\triangle}{=} A_D^\top P_o + P_o A_D - P_o L_D C_D - (P_o L_D C_D)^\top. \tag{17}$$

Rearranging entries and scaling corresponding columns and rows related to $H_i, i \in \mathscr{I}$, on the left side of the matrix in (15b), one obtains

$$P_c > 0, \quad P_o > 0, \tag{18a}$$

$$\begin{bmatrix} W_C & H_1^\top & \dots & H_N^\top & -P_c B_D K_D & P_c \\ H_1 & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N & 0 & \dots & -\gamma_N I & 0 & 0 \\ -(P_c B_D K_D)^\top & 0 & 0 & 0 & W_O & P_o \\ P_c & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0, \tag{18b}$$

where $\gamma_i = \frac{1}{\alpha_i^2} > 0$. Now the problem of stabilization of the large-scale system (1) by decentralized output feedback control is transferred to the problem of finding $\gamma_i > 0, i \in \mathscr{I}$, such that the inequalities in (18) are satisfied. Further, if the following optimization problem

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \quad \text{subject to (18)} \tag{19}$$

is feasible, the selection of the control gain matrix K_D and observer gain matrix L_D not only stabilizes the overall system (7) but also simultaneously maximizes the interconnection bounds α_i.

In the optimization problem given by (19), variables are P_c, P_o, K_D, L_D and $\gamma_i, i \in \mathscr{I}$. Since there are coupled term of matrix variables P_c and K_D, and P_o and L_D in the matrix inequality (18b), the optimization problem (19) results in a bilinear matrix inequality (Safonov *et al.*, 1994; Goh *et al.*, 1994) as opposed to an LMI. One has to find a way to transform the inequality (18b) to a form which is affine in the unknown variables. To achieve this, one can introduce variables

$$M_D \stackrel{\triangle}{=} P_c B_D K_D, \quad N_D \stackrel{\triangle}{=} P_o L_D. \tag{20}$$

Then, the optimization problem (19) becomes

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \quad \text{subject to } P_c > 0, \quad P_o > 0, \tag{21a}$$

$$\begin{bmatrix} W_C & H_1^\top & \dots & H_N^\top & -M_D & P_c \\ H_1 & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N & 0 & \dots & -\gamma_N I & 0 & 0 \\ -M_D^\top & 0 & 0 & 0 & W_O & P_o \\ P_c & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0. \tag{21b}$$

The solution to the optimization problem (21) gives rise to M_D and N_D. The controller and observer gain matrices were obtained from M_D and N_D in Siljak & Stipanovic (2001) in the following manner. The observer gain matrix L_D can be computed using (20) as

$$L_D = P_o^{-1} N_D.$$

However, controller gain matrix K_D can be obtained only in the case when B_D is invertible, i.e.

$$K_D = B_D^{-1} P_c^{-1} M_D.$$

Obviously, invertibility of B_D requires that $B_i, i \in I$, be invertible, which is too restrictive. When all the B_i are not invertible, it is not possible to obtain the control gain matrix K_D from the optimization problem (21). The following addresses the proposed LMI solution to the case when B_i is not invertible.

One can pre-multiply and post-multiply the left-hand side of (18b) by $\operatorname{diag}(P_c^{-1}, I)$ and define $Y = P_c^{-1}$ to obtain following conditions which are equivalent to (18):

$$Y > 0, \quad P_o > 0, \tag{22a}$$

$$\begin{bmatrix} W_C' & YH_1^\top & \dots & YH_N^\top & -B_D K_D & I \\ H_1 Y & -\gamma_1 I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ H_N Y & 0 & \dots & -\gamma_N I & 0 & 0 \\ -(B_D K_D)^\top & 0 & 0 & 0 & W_O & P_o \\ I & 0 & 0 & 0 & P_o & -I \end{bmatrix} < 0, \tag{22b}$$

where

$$W_C' \triangleq Y A_D^\top + A_D Y + (B_D K_D Y)^\top + (B_D K_D Y). \tag{23}$$

Let $\bar{M}_D \triangleq K_D Y$,

$$\begin{bmatrix} S_1 & S_2 \end{bmatrix} \triangleq \begin{bmatrix} -B_D K_D & I \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad F_c \triangleq \begin{bmatrix} W_C' & YH_1^\top & \dots & YH_N^\top \\ H_1 Y & -\gamma_1 I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_N Y & 0 & \dots & -\gamma_N I \end{bmatrix} < 0, \quad F_o \triangleq \begin{bmatrix} W_O & P_o \\ P_o & -I \end{bmatrix} < 0.$$

With these definitions, the problem now is to find Y, P_o, K_D, L_D and $\gamma_i, i \in \mathscr{I}$, from the following optimization problem:

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \quad \text{subject to } Y > 0, \quad P_o > 0,$$
$$\begin{bmatrix} F_c & S_1 & S_2 \\ S_1^T & W_O & P_o \\ S_2^T & P_o & -I \end{bmatrix} < 0. \tag{24}$$

We propose to solve this optimization problem by the following two steps.

Step 1: Maximize the interconnection bounds α_i $(= 1/\gamma_i)$ by solving the following optimization problem:

$$\text{Minimize} \sum_{i=1}^{N} \gamma_i \quad \text{subject to } Y > 0, \quad F_c < 0. \tag{25}$$

The optimization problem (25) gives Y and $\bar{M}_D$. The control gain K_D is obtained from Step 1 as

$$K_D = \bar{M}_D Y^{-1}. \tag{26}$$

Step 2: Using the K_D obtained from Step 1, find P_o and N_D by solving the following optimization problem:

$$\text{Minimize} \sum_{i=1}^{N} \beta_i \quad \text{subject to } P_o > 0, \quad \Lambda > 0,$$
$$\begin{bmatrix} \Lambda F_c & S_1 & S_2 \\ S_1^\top & W_O & P_o \\ S_2^\top & P_o & -I \end{bmatrix} < 0, \tag{27}$$

where $\Lambda = \text{diag}(\beta_1 I_1, \beta_2 I_2, \ldots, \beta_N I_N, \beta_1 I_1, \beta_2 I_2, \ldots, \beta_N I_N)$, I_i denotes the $n_i \times n_i$ identity matrix, $W_O \triangleq A_D^\top P_o + P_o A_D - N_D C_D - (N_D C_D)^\top$ and $N_D = P_o L_D$. The matrices F_c and S_1 in Step 2 are obtained from Step 1. The observer gain L_D is obtained as

$$L_D = P_o^{-1} N_D. \tag{28}$$

REMARK 1 Unlike the case when B_D is invertible, the LMIs given by (25) and (27) cannot be solved simultaneously. The optimization problem (25) of Step 1 must be solved followed by Step 2.

REMARK 2 Since Y, A_D, B_D and $\bar{M}_D$ are all block diagonal matrices, we can conclude that $\Lambda F_c = \Lambda^{1/2} F_c \Lambda^{1/2} < 0$ when $F_c < 0$. Also, if $\beta_i > 1, i \in \mathscr{I}$, $\Lambda F_c < F_c < 0$. Further, note that the solution K_D obtained from the optimization problem $F_c < 0$ is unchanged if we solve the optimization problem $\Lambda F_c < 0$ because of the chosen structure of Λ.

REMARK 3 If $\Lambda = I$, the LMI (27) may not be feasible for the selection of F_c and K_D resulting from the optimization problem (25). On the other hand, by choosing Λ as a matrix variable, the LMI (27) becomes feasible. The feasibility of the LMI problems (25) and (27) is given by the following lemma.

LEMMA 1 The optimization problems given by (25) and (27) are feasible if the pairs (A_i, B_i) and (A_i, C_i) are controllable and observable, respectively, for all $i \in \mathscr{I}$.

Proof. To prove the LMI optimization problem (25) is feasible, one needs to show that there exists a solution that satisfies the inequality (25). In view of (25) and H_i being constant matrices, to show that there exist $Y > 0$, $\bar{M}_D$, $\gamma_i > 0$, such that $F_c < 0$, it is sufficient to show that

$$\text{there exists } Y > 0, \bar{M}_D \text{ such that } W_C' < 0. \tag{29}$$

Note that

$$\begin{aligned} W_C' &= Y A_D^\top + A_D Y + (B_D \bar{M}_D)^\top + B_D \bar{M}_D \\ &= P_c^{-1} A_D^\top + A_D P_c^{-1} + (B_D K_D P_c^{-1})^\top + B_D K_D P_c^{-1} \\ &= P_c^{-1}((A_D + B_D K_D)^\top P_c + P_c(A_D + B_D K_D)) P_c^{-1}. \end{aligned}$$

Since (A_i, B_i) is a controllable pair (which implies (A_D, B_D) is a controllable pair), there exist a $P_c > 0$ and a K_D such that

$$(A_D + B_D K_D)^\top P_c + P_c(A_D + B_D K_D) = -Q_D < 0$$

for any positive definite matrix Q_D. Now, using the Schur complement lemma, we have

$$F_c \triangleq \begin{bmatrix} W_C' & R^T \\ R & \Gamma_\gamma \end{bmatrix} < 0 \iff \Gamma_\gamma < 0 \quad \text{and} \quad W_C' - R^T \Gamma_\gamma^{-1} R < 0, \tag{30}$$

where $R^T = [Y H_1^T \; Y H_2^T \; \cdots \; Y H_N^T]$ and $\Gamma_\gamma = \operatorname{diag}(-\gamma_1 I, -\gamma_2 I, \ldots, -\gamma_N I)$. Therefore, from the last condition of (30), $F_c < 0$ is guaranteed by the existence of a large enough Γ_γ. □

For the optimization problem (27),

$$\begin{bmatrix} \Lambda F_c & S \\ S^T & F_o \end{bmatrix} < 0 \iff \Lambda F_c < 0 \quad \text{and} \quad F_o - S(\Lambda F_c)^{-1} S^T < 0, \tag{31}$$

where $S = [S_1 \; S_2]$. Therefore, to show feasibility of this problem it is sufficient to show that a positive definite solution P_o exists to the problem $F_o < 0$. Using the same arguments as given earlier, observability of the pair (A_i, C_i), for all $i \in I$, implies existence of a positive definite solution to the problem $F_o < 0$.

REMARK 4 The final uncertainty gains are $\beta_i \gamma_i$, $i \in \mathscr{I}$, where γ_i is obtained from the optimization problem (25) and β_i is obtained from (27).

The LMI optimization problems given by (25) and (27) do not pose any restrictions on the size of the matrix variables Y, $\bar{M}_D$, P_o and N_D. Consequently, the results of these two optimization problems may yield very large controller and observer gain matrices K_D and L_D, respectively. In view of (26) and (28), one can restrict K_D and L_D by posing constraints on the matrices Y, $\bar{M}_D$, P_{o} and N_D, and a further constraint on γ_i (Siljak & Stipanovic, 2000) as

$$\begin{gathered}\gamma_i - \frac{1}{\bar{\alpha}_i^2} < 0, \quad \bar{\alpha}_i > 0; \qquad {Y_i}^{-1} < \kappa_{Y_i} I, \quad \kappa_{Y_i} > 0; \\ \bar{M}_{D_i} \bar{M}_{D_i}^{\top} < \kappa_{\bar{M}_{D_i}} I, \quad \kappa_{\bar{M}_{D_i}} > 0;\end{gathered} \tag{32}$$

$$\begin{gathered}\beta_i - \bar{\beta}_i > 0, \quad \bar{\beta}_i > 0; \qquad {P_{\mathrm{o}_i}}^{-1} < \kappa_{P_{\mathrm{o}_i}} I, \quad \kappa_{P_{\mathrm{o}_i}} > 0; \\ N_{D_i}^{\top} N_{D_i} < \kappa_{N_{D_i}} I, \quad \kappa_{N_{D_i}} > 0,\end{gathered} \tag{33}$$

where $\bar{M}_{D_i}$ and N_{D_i} are the ith diagonal block of $\bar{M}_D$ and N_D, respectively. Equations (32) and (33) are, respectively, equivalent to

$$\gamma_i - \frac{1}{\bar{\alpha}_i^2} < 0, \quad \begin{bmatrix} -Y_i & -I \\ -I & -\kappa_{Y_i} I \end{bmatrix} < 0, \quad \begin{bmatrix} -\kappa_{\bar{M}_{D_i}} I & \bar{M}_{D_i} \\ \bar{M}_{D_i}^{\top} & -I \end{bmatrix} < 0, \quad \kappa_{Y_i}, \kappa_{\bar{M}_{D_i}} > 0, \tag{34}$$

$$\beta_i - \bar{\beta}_i > 0, \quad \begin{bmatrix} -P_{\mathrm{o}_i} & -I \\ -I & -\kappa_{P_{\mathrm{o}_i}} I \end{bmatrix} < 0, \quad \begin{bmatrix} -\kappa_{N_{D_i}} I & N_{D_i}^{\top} \\ N_{D_i} & -I \end{bmatrix} < 0, \quad \kappa_{N_{D_i}}, \kappa_{P_{\mathrm{o}_i}} > 0. \tag{35}$$

Combining (25) and (34), (27) and (35) and changing the optimization objectives to the minimization of $\sum_{i=1}^{N} \left(\gamma_i + \kappa_{Y_i} + \kappa_{\bar{M}_{D_i}}\right)$ and $\sum_{i=1}^{N} \left(\beta_i + \kappa_{P_{\mathrm{o}_i}} + \kappa_{N_{D_i}}\right)$, respectively, results in the following two LMI optimization problems:

Step 1′: Maximize the interconnection bounds α_i by solving the following optimization problem:

$$\text{Minimize} \sum_{i=1}^{N} \left(\gamma_i + \kappa_{Y_i} + \kappa_{\bar{M}_{D_i}}\right) \qquad \text{subject to (25) and (34).} \tag{36}$$

Step 2′: Find P_{o} and N_D by using K_D obtained from Step 1′ and solving the following optimization problem:

$$\text{Minimize} \sum_{i=1}^{N} \left(\beta_i + \kappa_{P_{\mathrm{o}_i}} + \kappa_{N_{D_i}}\right) \qquad \text{subject to (27) and (35).} \tag{37}$$

Similar to Lemma 1, it can be shown that the optimization problems (36) and (37) are feasible when all the subsystems are controllable and observable, provided that $\bar{\alpha}_i$ is chosen sufficient small; one can choose large $\bar{\beta}$, $\kappa_{\bar{M}_{D_i}}$, κ_{Y_i}, $\kappa_{N_{D_i}}$ and $\kappa_{P_{\mathrm{o}_i}}$ and small $\bar{\alpha}_i$ to satisfy (34) and (35).

The results of the LMI solution to the decentralized output feedback control problem for the large-scale system (1) are summarized in the following theorem.

THEOREM 1 Consider the large-scale system (1) with the observer given by (6) and the controller given by (5). If

$$\alpha_i \leqslant \min\left(\frac{1}{\sqrt{\gamma_i}}, \frac{1}{\sqrt{\beta_i \gamma_i}}\right), \tag{38}$$

where γ_i and β_i are solutions to the optimization problems (36) and (37), then the selection of controller and observer gain matrices given by (26) and (28) results in a stable closed-loop system.

3. Numerical example

Consider the following large-scale system composed of two subsystems:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + h_1(x), \quad y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1; \tag{39a}$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 + h_2(x), \quad y_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_2, \tag{39b}$$

where $x_1 = [x_{11}\ x_{12}]^\top$, $x_2 = [x_{21}\ x_{22}\ x_{23}]^\top$, $x = [x_1^\top\ x_2^\top]^\top$, $h_1(x) = \alpha_1 \cos(x_{22}) H_1 x$, $h_2(x) = \alpha_1 \cos(x_{11}) H_2 x$, $\alpha_1 = \alpha_2 = 0.1$,

$$H_1 = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad H_2 = \frac{1}{\sqrt{15}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Choosing $\bar{\alpha}_i = 0.001$, $\beta_i = 0.0001$, $i = 1, 2$, and solving the optimization problems (36) and (37) results in

$$M_D = \begin{bmatrix} 2.7773 & -3.0854 & 0 & 0 & 0 \\ 0 & 0 & -0.20276 & -1.6091 & -3.1217 \end{bmatrix},$$

$$Y = \begin{bmatrix} 25.4611 & -14.0572 & 0 & 0 & 0 \\ -14.0572 & 9.36744 & 0 & 0 & 0 \\ 0 & 0 & 47.4557 & -35.8544 & 6.51817 \\ 0 & 0 & -35.8544 & 41.4759 & -17.3584 \\ 0 & 0 & 6.51817 & -17.3584 & 14.7546 \end{bmatrix},$$

$$N_D = \begin{bmatrix} 0.65833 & 0 \\ 0.51052 & 0 \\ 0 & 1.193 \\ 0 & 0.98386 \\ 0 & -0.39479 \end{bmatrix},$$

$$P_o = \begin{bmatrix} 0.98889 & -0.39895 & 0 & 0 & 0 \\ -0.39895 & 0.59679 & 0 & 0 & 0 \\ 0 & 0 & 1.4986 & -0.29952 & -0.12104 \\ 0 & 0 & -0.29952 & 0.50936 & -0.43087 \\ 0 & 0 & -0.12104 & -0.43087 & 0.70333 \end{bmatrix},$$

$$\gamma_1 = 13.8634, \quad \gamma_2 = 2.7773, \quad \beta_1 = 4.9354, \quad \beta_2 = 11.3989.$$

Gain matrices K_D and L_D are found to be

$$K_D = \begin{bmatrix} -0.42433 & -0.96614 & 0 & 0 & 0 \\ 0 & 0 & -0.8614 & -1.404 & -1.4828 \end{bmatrix},$$

$$L_D = \begin{bmatrix} 1.3841 & 0 \\ 1.7807 & 0 \\ 0 & 2.4812 \\ 0 & 6.8017 \\ 0 & 4.0325 \end{bmatrix}$$

by using (26) and (28), respectively. It is easy to check that the condition given by (38) is satisfied. Hence, according to Theorem 1, the closed-loop system is quadratically stable.

The simulation results are shown in Figures 1 and 2. In Figure 1, the state x_{11} and its estimate $\widehat{x}_{11}$, the state x_{12} and its estimate $\widehat{x}_{12}$ and the control u_1 are shown in the first, second and third plot, respectively. Figure 2 shows the states x_2, their estimates $\widehat{x}_2$ and the control u_2. It can be observed from both the figures that the state of the overall system, x, and their estimates, $\widehat{x}$, converge to zero.

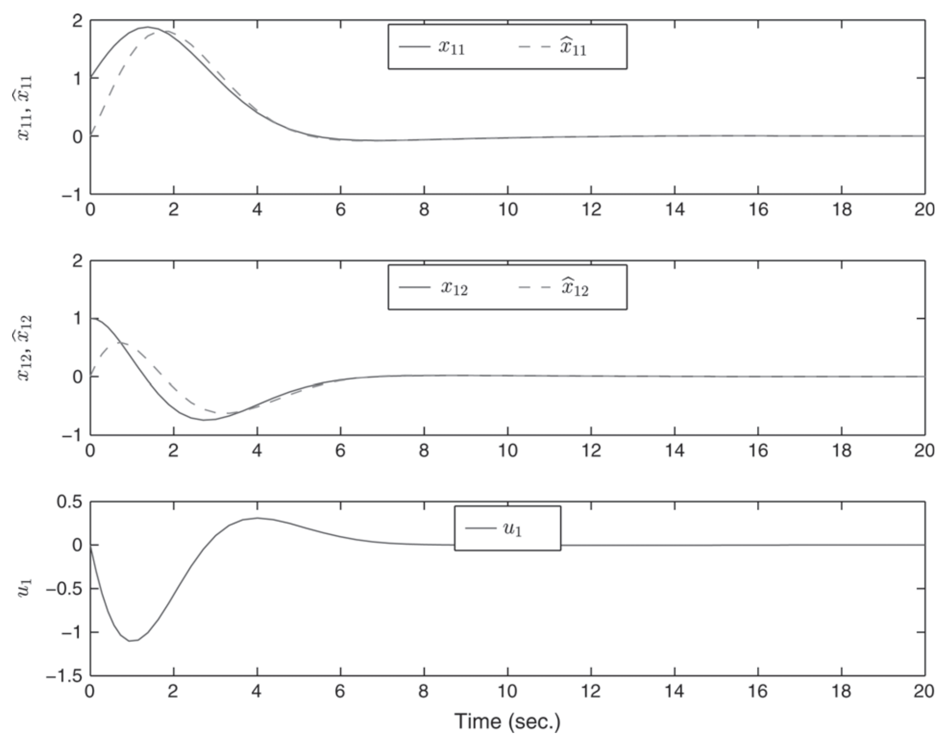

FIG. 1. Simulation result for the first subsystem (39a).

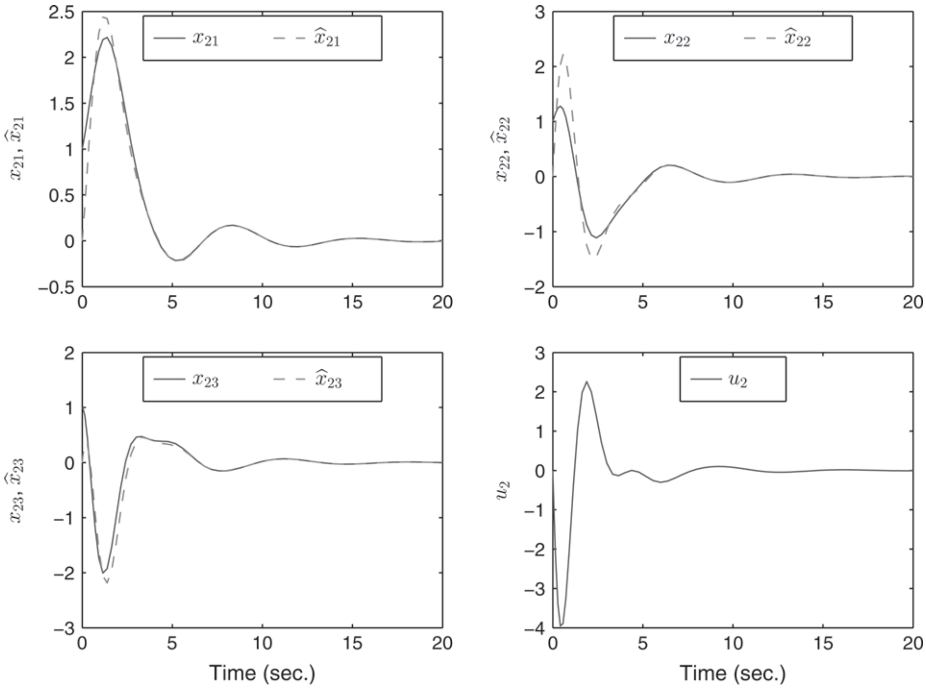

FIG. 2. Simulation result for the second subsystem (39b).

4. Summary

In this paper, an LMI solution to the decentralized output feedback control problem for a class of large-scale interconnected non-linear systems is given. The interconnecting non-linearity of each subsystem was assumed to be bounded by a quadratic form of states of the overall system. Local output signals from each subsystem were used to generate the local control inputs and exact knowledge of the non-linear interconnections is not required for the proposed solution. Simulation results on a numerical example verify the proposed design. The contribution of this research over prior work is that the requirement that the input matrix of each subsystem be invertible is relaxed.

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REFERENCES

ABDEL-JABBAR, N., KRAVARIS, C. & CARNAHAN, B. (1998) A partially decentralized state observer and its parallel computer implementation. *Indust. Eng. Chem. Res.*, **37**, 2741–2760.

ALDEEN, M. & MARSH, J. F. (1999) Decentralised observer-based control scheme for interconnected dynamical systems with unknown inputs. *IEE Proc. Control Theory Appl.*, **146**, 349–357.

BOYD, S., GHAOUI, L. E., FERON, E. & BALAKRISHNAN, V. (1994) *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM Studies in Applied Mathematics.

GOH, K. C., TURAN, L., SAFONOV, M. G., PAPAVASSILOPOULOS, G. P. & LY, J. H. (1994) Biaffine matrix inequality properties and computational methods. *Proceedings of the American Control Conference*. Baltimore, MD.

GONG, Z. (1995) Decentralized robust control of uncertain interconnected systems with prescribed degree of exponential convergence. *IEEE Trans. Autom. Control*, **40**, 704–707.

IKEDA, M. (1989) Decentralized control of large scale systems. *Three Decades of Mathematical System Theory*. Springer, pp. 219–242.

IOANNOU, P. A. (1986) Decentralized adaptive control of interconnected systems. *IEEE Trans. Autom. Control*, **31**, 291–298.

JIANG, Z. P. (2000) Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback. *IEEE Trans. Autom. Control*, **45**, 2122–2128.

MIRKIN, B. M. & GUTMAN, P. O. (2003) Decentralized output-feedback mrac of linear state delay systems. *IEEE Trans. Autom. Control*, **48**, 1613–1619.

NARENDRA, K. S. & OLENG, N. O. (2002) Exact output tracking in decentralized adaptive control systems. *IEEE Trans. Autom. Control*, **47**, 390–395.

PAGILLA, P. R. & ZHU, Y. (2005) A decentralized output feedback controller for a class of large-scale interconnected nonlinear systems. *ASME J. Dynam. Syst. Meas. Control*, **127**, 167–172.

SAFONOV, M. G., GOH, K. C. & LY, J. H. (1994) Control system synthesis via bilinear matrix inequalities. *Proceedings of the American Control Conference*. Baltimore, MD.

SANDELL, J. N. R., VARAIYA, P., ATHANS, M. & SAFANOV, M. G. (1978) Survey of decentralized control methods for large scale systems. *IEEE Trans. Autom. Control*, **23**, 108–128.

SILJAK, D. D. (1991) *Decentralized Control of Complex Systems*. New York: Academic Press.

SILJAK, D. D. & STIPANOVIC, D. M. (2000) Robust stabilization of nonlinear systems: the LMI approach. *Math. Probl. Eng.*, **6**, 461–493.

SILJAK, D. D. & STIPANOVIC, D. M. (2001) Autonomous decentralized control. *Proceedings of the International Mechanical Engineering Congress and Exposition*. Nashville, TN.

SILJAK, D. D., STIPANOVIC, D. M. & ZECEVIC, A. I. (2002) Robust decentralized turbine/governor control using linear matrix inequalities. *IEEE Trans. Power Syst.*, **17**, 715–722.

VISWANADHAM, N. & RAMAKRISHNA, A. (1982) Decentralized estimation and control for interconnected systems. *Large Scale Syst.*, **3**, 255–266.

ZHANG, S. Y., MIZUKAMI, K. & WU, H. S. (1996) Decentralized robust control for a class of uncertain -scale interconnected nonlinear dynamical systems. *J. Optim. Theory Appl.*, **91**, 235–256.