Adaptive Controller and Observer Design for a Class of Nonlinear Systems

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Design of a stable adaptive controller and observer for a class of nonlinear systems that contain product of unmeasurable states and unknown parameters is considered. The nonlinear system is cast into a suitable form based on which a stable adaptive controller and observer are designed using a parameter dependent Lyapunov function. The class of nonlinear systems considered is practically relevant; mechanical systems with dynamic friction fall into this category. Experimental results on a single-link mechanical system with dynamic friction are shown for the proposed design. [DOI: 10.1115/1.2234489]

1 Introduction

Many practical applications require estimation of the states and parameters that can be used in designing a stable control algorithm. The unmeasurable states and parameters are generally estimated based on available measurements and the knowledge of the physical system, such as a model. Design of a stable adaptive observer that simultaneously estimates the unmeasurable state and the unknown parameters for a general class of nonlinear systems is a challenging problem; often one considers special classes of systems to arrive at satisfactory designs. This has led to continued strong interest over the years in the development of stable adaptive controllers and observers.

Early work on stable adaptive observers for linear time-invariant systems can be found in [1,2]. A large number of results in adaptive control of linear and nonlinear systems can be found in [3–5]. In [6], an extensive survey of adaptive output feedback control methods for nonlinear systems without derived input signal measurements was given. A new method for the design of locally convergent observers using the backstepping method was proposed in [7]. A discussion of persistent excitation in adaptive systems was given in [8]. In [9], design of a nonlinear observer for nonlinear systems using Lyapunov's auxiliary theorem was proposed. In [10–12], adaptive observers were designed for a class of nonlinear systems, which can be transformed into an adaptive observer canonical form, where the unknown parameters appear linearly in functions of known output and input variables only.

In [11], the linear adaptive observer was extended to a class of nonlinear systems that can be transformed into an adaptive observer canonical form given by

$$\dot{x} = Mx + \psi_0(t) + \psi_1(y,u)\theta(t), \qquad (1a)$$

$$y = x_1 \qquad (1b)$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}$ are the state vector and the output of the system, respectively; x_1 is the first element of x; $M \in \mathbb{R}^{n\times n}$ is a known constant matrix; $\theta(t) \in \mathbb{R}^p$ is an unknown parameter vector; $\psi_0(t) \in \mathbb{R}^n$ is a known function vector; and $\psi_1(y,u) \in \mathbb{R}^{n\times p}$ is a known matrix of known functions of u and y. Note that the system in (1) is linear in the unknown parameters and the nonlinearities are restricted to be functions of the known input and output signals. An adaptive observer, which is driven by a $p(n-1)$ dimensional auxiliary filter, was developed for (1); stable convergence of the estimates was shown under certain persistency of excitation conditions.

Necessary and sufficient conditions for transforming a general nonlinear system into a canonical form that is nonlinear purely in the output variables can be found in [12]. Based on the early work of [11,10], extensions on adaptive nonlinear observers was reported by Marino et al. in a series of papers; see [13] and the references therein; Marino et al. studied adaptive observers for a class of nonlinear systems that can be transformed via a global state space diffeomorphism into

$$\dot{x} = Mx + \psi_0(y,u) + b\psi_1^{\mathsf{T}}(y,u)\theta, \qquad (2a)$$

$$y = x_1 \qquad (2b)$$

where $\psi_0(y,u) \in \mathbb{R}^n$ and $\psi_1(y,u) \in \mathbb{R}^p$ are known smooth functions of u and y, $b \in \mathbb{R}^n$ is a constant known vector, and

$$M = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix},$$

where I_{n-1} denotes the $(n-1)\times(n-1)$ dimensional identity matrix. Note that the system is linear in the unknown parameters and the nonlinearities are functions of the known output and input variables only. In [14], an output feedback controller for a class of nonlinear systems that consist of a set of unknown constant parameters and unmeasurable state variables was considered. Under the assumption that the dynamics of the unmeasurable state is asymptotically stable and using parameter dependent filtered transformations, an output feedback controller was constructed. A necessary and sufficient condition to transform a given nonlinear system to the form given by (2) was given in [15]; it was shown that the controllability and observability properties of the system are invariant under the given transformation.

In this paper, we design a new adaptive controller and observer for a class of nonlinear systems that contain products of unmeasurable states and unknown parameters. We cast the given nonlinear system into a suitable modified form that facilitates the proposed design procedure; the process of casting a given system into the modified form is constructive and is always possible. Stability of the proposed controller and observer is shown using a parameter dependent Lyapunov function. Experimental results on a single-link mechanical system are shown to verify the proposed design.

The rest of the paper is organized as follows. The problem statement, including the class of systems, assumptions, and control objective, is given in Sec. 2. In Sec. 3 we give the procedure for obtaining the modified form of the dynamics of the original system. Based on the modified form of the dynamics, the adaptive controller and observer design are presented in Sec. 4. Experimental results are given in Sec. 5. Conclusions and future research are given in Sec. 6.

2 Problem Statement

We consider the following class of systems that contain the product of the unmeasurable state variables and unknown parameters:

$$\dot{x} = Mx + hu + h[d(x) + f_\theta^{\mathsf{T}}(x)\theta + f_z^{\mathsf{T}}(x)z + \theta^{\mathsf{T}}G_z(x)z], \qquad (3a)$$

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$$\dot{z} = B_z(x)z + a_z(x) \tag{3b}$$

where $x \in \mathbb{R}^n$ is the measured state, $z \in \mathbb{R}^m$ is the unmeasured state, $u \in \mathbb{R}$ is the control input, $\theta \in \mathbb{R}^p$ is an unknown constant parameter vector, $M \in \mathbb{R}^{n\times n}$ is a known constant matrix, $h \in \mathbb{R}^n$ is a known constant vector, and $d(x) \in \mathbb{R}$, $f_\theta(x) \in \mathbb{R}^p$, $f_z(x) \in \mathbb{R}^m$, $G_z(x) \in \mathbb{R}^{p\times m}$, $a_z(x) \in \mathbb{R}^m$, and $B_z(x) \in \mathbb{R}^{m\times m}$ are known smooth functions of x. Notice that the last term in the x dynamics in (3) is a product of the unmeasurable state z and the unknown parameter vector θ. We assume that the system dynamics described by (3) satisfy the following:

(1) The pair (M,h) is controllable.

(2) There exists a symmetric positive definite matrix $P_z \in \mathbb{R}^{m\times m}$ such that $B_z^{\mathsf{T}}(x)P_z + P_z B_z(x) \leqslant -Q_z$ for all $x \in \mathbb{R}^n$, where Q_z is a positive semidefinite matrix. Also, for every bounded $x(t)$, the solution $z(t)$ is bounded for any initial condition $z(t_0)$.

(3) The sign of each parameter, θ_i, $i=1:p$, in the parameter vector θ is known, and θ_i is bounded.

(4) The functions $d(x)$, $f_\theta(x)$, $f_z(x)$, $G_z(x)$, $a_z(x)$, and $B_z(x)$ are bounded functions of x.

Assumption (1) guarantees the existence of a control gain vector $c \in \mathbb{R}^n$ that can stabilize the linear part of the x dynamics; that is, there exists a symmetric positive definite solution, P, to the Lyapunov equation $(M-hc^{\mathsf{T}})^{\mathsf{T}}P + P(M-hc^{\mathsf{T}}) = -Q$, where Q is a symmetric positive definite matrix and c is a feedback gain vector. Assumption (2) ensures that z has a stable dynamics provided $x(t)$ is bounded. Assumption (3) is the only required knowledge of the unknown parameters and is reasonable for many practical plants; it allows us to use a parameter dependent Lyapunov function candidate during the design process.

Our objective in the paper is to design an adaptive controller and observer such that asymptotic regulation of the measurable state, asymptotic convergence of the state estimation errors to zero or their boundedness based on certain conditions, and boundedness of the estimated parameters are achieved. Instead of using the parameter dependent filtered transformations available in literature, the proposed approach is to cast the system in a modified form that can be used to design a control algorithm based on a parameter dependent Lyapunov function. The process of casting the original nonlinear system into the modified form is constructive and is always possible.

3 Modified Form of the System Dynamics

In this section, the procedure of expressing the system given by (3) into a modified form will be described. The nonlinear system (3) can be cast in the following form:

$$\dot{x} = Mx + h[u + d(x) + f^{\mathsf{T}}(x)\Theta + Z^{\mathsf{T}}G(x)\Theta], \tag{4a}$$

$$\dot{Z} = a(x) + B(x)Z \tag{4b}$$

where

$$Z^{\mathsf{T}} = [\underbrace{z^{\mathsf{T}}, \ldots, z^{\mathsf{T}}, \ldots, z^{\mathsf{T}}}_{(p+1)\text{-times}}],$$

$$\Theta^{\mathsf{T}} = [\underbrace{\theta_1, \ldots, \theta_1}_{m\text{-times}}, \underbrace{\theta_2, \ldots, \theta_2}_{m\text{-times}}, \ldots, \underbrace{\theta_p, \ldots, \theta_p}_{m\text{-times}}, \underbrace{1, \ldots, 1}_{m\text{-times}}],$$

$$f^{\mathsf{T}}(x) = [f_{\theta_1}(x), \underbrace{0, \ldots, 0}_{(m-1)\text{-times}}, \ldots, f_{\theta_p}(x), \underbrace{0, \ldots, 0}_{(m-1)\text{-times}}, \underbrace{0, \ldots, 0}_{m\text{-times}}],$$

$$a^{\mathsf{T}}(x) = [\underbrace{a_z^{\mathsf{T}}(x), \ldots, a_z^{\mathsf{T}}(x), \ldots, a_z^{\mathsf{T}}(x)}_{(p+1)\text{-times}}],$$

$$B(x) = \mathrm{diag}(\underbrace{B_z(x), \ldots, B_z(x), \ldots, B_z(x)}_{(p+1)\text{-times}}),$$

$$G(x) = \mathrm{diag}(g_{z11}(x), \ldots, g_{z1m}(x), \ldots, g_{zp1}(x), \ldots, g_{zpm}(x), f_{z1}(x), \ldots, f_{zm}(x))$$

where $\mathrm{diag}(\cdot)$ denotes the diagonal matrix or block diagonal matrix, $g_{zij}(x)$, $i=1:p$, $j=1:m$, is the ijth element of $G_z(x)$, and $f_{zi}(x)$, $i=1:m$, is the ith element of $f_z(x)$, $f_{\theta i}$, $i=1:p$, is the ith element of $f_\theta(x)$.

Equations (3) and (4) describe the same system. The unknown parameter vector Θ and the unmeasurable state vector Z in (4) are of a larger dimension than that of θ and z in (3); $Z \in \mathbb{R}^{m(p+1)}$ is a vector cascaded by $(p+1)$ z's; $\Theta \in \mathbb{R}^{m(p+1)}$ is a vector cascaded by m times θ_i's, $i=1:p$, and an m-vector with each entry equal to 1; $a(x) \in \mathbb{R}^{m(p+1)}$ is cascaded by $(p+1)$ $a_z(x)$'s; $G(x) \in \mathbb{R}^{m(p+1)\times m(p+1)}$ is a diagonal matrix whose diagonal entries are the entries of $G_z(x)$ and $f_z(x)$; $f(x) \in \mathbb{R}^{m(p+1)}$ is a vector whose $[(j-1)m+1]$th element is $f_{\theta j}(x)$, $j=1:p$ and the other elements equal to zero.

The motivation for casting the nonlinear system described by (3) in the form given by (4) with a new parameter vector Θ and a new state vector Z is to account for the nonzero off-diagonal entries in $G_z(x)$ and the nonzero entries in $f_z(x)$; as a result of this, the proposed stable adaptive controller and observer design is feasible. A nonzero off-diagonal entry in $G_z(x)$ means that two unknown parameters are coupled with the same unmeasurable state variable (or two unmeasurable state variables are coupled with the same parameter). Assuming that none of the elements of the matrix $G_z(x)$ and vector $f_z(x)$ are zero, then $Z \in \mathbb{R}^q$ and $\Theta \in \mathbb{R}^q$, where $q=m(p+1)$. If some entries in $G_z(x)$ and/or $f_z(x)$ are zero, it is possible to reduce the dimension of the vector Z. Correspondingly, the dimensions of $G(x)$, Θ, $f(x)$, $a(x)$, and $B(x)$ are also reduced.

The following example will illustrate the reduction procedure. Consider the system described by (3) with $m=2$, $p=3$, and $B_z(x)$ diagonal. The modified system has

$$Z^{\mathsf{T}} = [z_1, z_2, z_1, z_2, z_1, z_2, z_1, z_2],$$

$$\Theta^{\mathsf{T}} = [\theta_1, \theta_1, \theta_2, \theta_2, \theta_3, \theta_3, 1, 1],$$

$$G(x) = \mathrm{diag}[g_{z11}(x), g_{z12}(x), g_{z21}(x), g_{z22}(x), g_{z31}(x), g_{z32}(x), f_{z1}(x), f_{z2}(x)],$$

$$a^{\mathsf{T}}(x) = [a_{z1}(x), a_{z2}(x), a_{z1}(x), a_{z2}(x), a_{z1}(x), a_{z2}(x), a_{z1}(x), a_{z12}(x)],$$

$$B(x) = \mathrm{diag}[B_z(x), B_z(x), B_z(x), B_z(x)],$$

$$f^{\mathsf{T}}(x)=[f_{\theta 1}(x),0,f_{\theta 2}(x),0,f_{\theta 3}(x),0,0,0].$$

If $f_z^{\mathsf{T}}(x)=[0,0]$, discard the last two rows of Z, Θ, $a(x)$ and $f(x)$, and the last two rows and columns of $G(x)$ and $B(x)$, which will result in $q=6$, which is less than the maximum size of eight. If $g_{z12}(x)=0$, then the second row of Z, Θ, $a(x)$ and $f(x)$, and the second row and the second column of $G(x)$ and $B(x)$ can be discarded, which gives $q=7$.

With the assumptions on the original system described by (3), it is easy to see that the following three assumptions, which correspond to assumptions (2), (3), and (4), respectively, are true for the system in the modified form (4):

- (2′) There exists a symmetric positive definite matrix $P_Z=\operatorname{diag}(P_z,\ldots,P_z,\ldots,P_z)$ such that $B^{\mathsf{T}}(x)P_Z+P_ZB(x)\leqslant -Q_Z$. Also, for every bounded $x(t)$, the solution of $Z(t)$ is bounded for any initial condition $Z(t_0)$.
- (3′) The sign of each parameter, Θ_i, $i=1:q$, in the parameter vector Θ is known, and Θ_i is bounded.
- (4′) The functions $d(x)$, $f(x)$, $G(x)$, $a(x)$, and $B(x)$ are bounded functions of x.

4 Adaptive Controller and Observer Design

The following theorem illustrates the main result of this paper.

THEOREM 1. *Consider the plant described by (4), the following control law (5), parameter estimation algorithm (6) to estimate the constant parameter vector* Θ, *and observer* (7)

$$u=-c^{\mathsf{T}}x-d(x)-f^{\mathsf{T}}(x)\hat{\Theta}-\hat{Z}^{\mathsf{T}}G(x)\hat{\Theta}, \tag{5}$$

$$\dot{\hat{\Theta}}=2\Gamma[G(x)\hat{Z}+f(x)]h^{\mathsf{T}}Px, \tag{6}$$

$$\dot{\hat{Z}}=a(x)+B(x)\hat{Z}+P_Z^{-1}G(x)\operatorname{sgn}(\Theta)h^{\mathsf{T}}Px \tag{7}$$

where $\Gamma=\Gamma^{\mathsf{T}}\in\mathbb{R}^{q\times q}>0$, $c\in\mathbb{R}^n$, $\operatorname{sgn}(\Theta)=[\operatorname{sgn}(\Theta_1),\ldots,\operatorname{sgn}(\Theta_q)]^{\mathsf{T}}$, $(\hat{*})$ *denotes the estimate of* $(*)$ and $(\tilde{*})=(\hat{*})-(*)$ *denotes the estimation error of* $(*)$, *and c is chosen such that P is the symmetric positive definite solution of the Lyapunov equation*

$$(M-hc^{\mathsf{T}})^{\mathsf{T}}P+P^{\mathsf{T}}(M-hc^{\mathsf{T}})=-Q, \tag{8}$$

for any given positive definite matrix Q. Then, the closed-loop system has the following properties.

- (i) $u(t)$, $\hat{\Theta}(t)$, $\tilde{\Theta}(t)$, $\hat{Z}(t)$, *and* $\tilde{Z}(t)$ *are bounded.*
- (ii) $\lim_{t\to\infty}x(t)=0$.
- (iii) *If* $Q_z>0$, $\lim_{t\to\infty}\tilde{Z}(t)=0$.

Proof: Using the control input and the observer given by (5) and (7), respectively, the x dynamics and the state estimation error dynamics are

$$\dot{x}=(M-hc^{\mathsf{T}})x-h[f^{\mathsf{T}}(x)+\hat{Z}^{\mathsf{T}}G(x)]\tilde{\Theta}-h\Theta^{\mathsf{T}}G(x)\tilde{Z}, \tag{9}$$

$$\dot{\tilde{Z}}=B(x)\tilde{Z}+P_Z^{-1}G(x)\operatorname{sgn}(\Theta)h^{\mathsf{T}}Px. \tag{10}$$

Consider the following Lyapunov function candidate:

$$V(x,\tilde{\Theta},\tilde{Z},\Theta)=x^{\mathsf{T}}Px+\tfrac{1}{2}\tilde{\Theta}^{\mathsf{T}}\Gamma^{-1}\tilde{\Theta}+\tilde{Z}^{\mathsf{T}}\Lambda_{|\Theta|}P_Z\tilde{Z} \tag{11}$$

where $\Lambda_{|\Theta|}$ is a diagonal matrix whose ith diagonal element is the absolute value of the ith element of the parameter vector Θ, that is, $\Lambda_{|\Theta|}=\operatorname{diag}(|\theta_1|I_m,\ldots,|\theta_i|I_m,\ldots,|\theta_p|I_m,I_m)$. Notice that $V(x,\tilde{\Theta},\tilde{Z},\Theta)$ is indeed a Lyapunov function candidate because $\Lambda_{|\Theta|}P_Z$ is a symmetric positive definite matrix, which can be seen from the following:

$$\begin{aligned}\Lambda_{|\Theta|}P_Z&=\operatorname{diag}(|\theta_1|I_m,\ldots,|\theta_i|I_m,\ldots,|\theta_p|I_m,I_m)\\&\quad\times\operatorname{diag}(P_z,\ldots,P_z,\ldots,P_z)\\&=\operatorname{diag}(|\theta_1|P_z,\ldots,|\theta_i|P_z,\ldots,|\theta_p|P_z,P_z).\end{aligned} \tag{12}$$

Since P_z is a symmetric positive definite matrix, from (12) we can see that $\Lambda_{|\Theta|}P_Z$ is a symmetric positive definite matrix.

Taking the time derivative of $V(x,\tilde{\Theta},\tilde{Z},\Theta)$, and simplifying using (6), (9), and (10), we obtain

$$\begin{aligned}\dot{V}&=\dot{x}^{\mathsf{T}}Px+x^{\mathsf{T}}P\dot{x}+\tilde{\Theta}^{\mathsf{T}}\Gamma^{-1}\dot{\tilde{\Theta}}+2\tilde{Z}^{\mathsf{T}}\Lambda_{|\Theta|}P_Z\dot{\tilde{Z}}\\&=x^{\mathsf{T}}[(M-hc^{\mathsf{T}})^{\mathsf{T}}P+P(M-hc^{\mathsf{T}})]x+\tilde{Z}^{\mathsf{T}}[B^{\mathsf{T}}(x)P_Z\Lambda_{|\Theta|}\\&\quad+\Lambda_{|\Theta|}P_ZB(x)]\tilde{Z}+2\tilde{Z}^{\mathsf{T}}\Lambda_{|\Theta|}G(x)\operatorname{sgn}(\Theta)h^{\mathsf{T}}Px\\&\quad-2\tilde{Z}^{\mathsf{T}}G(x)\Theta h^{\mathsf{T}}Px\end{aligned} \tag{13}$$

Since $G(x)$ and $\Lambda_{|\Theta|}$ are diagonal, we have $\Lambda_{|\Theta|}G(x)\operatorname{sgn}(\Theta)=G(x)\Lambda_{|\Theta|}\operatorname{sgn}(\Theta)=G(x)\Theta$. Also,

$$\begin{aligned}&B^{\mathsf{T}}(x)P_Z\Lambda_{|\Theta|}+\Lambda_{|\Theta|}P_ZB(x)\\&\quad=\operatorname{diag}(|\theta_1|[B_z^{\mathsf{T}}(x)\ P_z+P_zB_z(x)],\ldots,|\theta_p|[B_z^{\mathsf{T}}(x)P_z+P_zB_z(x)]\\&\quad\leqslant\operatorname{diag}(-|\theta_1|Q_z,\ldots,-|\theta_p|Q_z)\\&\quad=-\Lambda_{|\Theta|}Q_Z\end{aligned}$$

where $Q_Z\triangleq\operatorname{diag}(Q_z,\ldots,Q_z)$. Notice that $\Lambda_{|\Theta|}Q_Z$ is a positive semidefinite matrix. Therefore, we have

$$\dot{V}\leqslant-x^{\mathsf{T}}Qx-\tilde{Z}^{\mathsf{T}}\Lambda_{|\Theta|}Q_Z\tilde{Z}\leqslant-\lambda_{\min}(Q)x^{\mathsf{T}}x-\lambda_{\min}(\Lambda_{|\Theta|}Q_Z)\tilde{Z}^{\mathsf{T}}\tilde{Z} \tag{14}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix.

Hence, (11) is a Lyapunov function for the closed-loop system, which guarantees that x, $\tilde{\Theta}$, and $\tilde{Z}$ are bounded; $\hat{\Theta}$ is bounded because $\hat{\Theta}=\tilde{\Theta}+\Theta$ and Θ is bounded; Z is bounded by assumptions (2) and (4), which in turn guarantees that $\hat{Z}$ $(=Z+\tilde{Z})$ is bounded; the control input $u(t)$ is bounded as it is a function of all bounded variables. From Eqs. (9) and (10), both $\dot{\tilde{Z}}$ and $\dot{x}$ are bounded. Therefore, $\tilde{Z}\in\mathcal{L}_\infty$, $\dot{\tilde{Z}}\in\mathcal{L}_\infty$, $x\in\mathcal{L}_\infty\cap\mathcal{L}_2$, and $\dot{x}\in\mathcal{L}_\infty$. By invoking Barbalat's Lemma [4], we obtain $\lim_{t\to\infty}x=0$. Moreover, if Q_z is positive definite, then $\tilde{Z}\in\mathcal{L}_2$; therefore, $\lim_{t\to\infty}\tilde{Z}=0$. ■

Remark 1. Theorem 1 addresses the regulation problem for the class of nonlinear systems described by (3). This design process can be extended to the tracking problem as well, which is shown in the example considered in the next section. Further, one can also extend the proposed design to multiple-input systems.

Remark 2. Notice that the original system described by (3) contains m state variables and p parameters that are to be estimated.

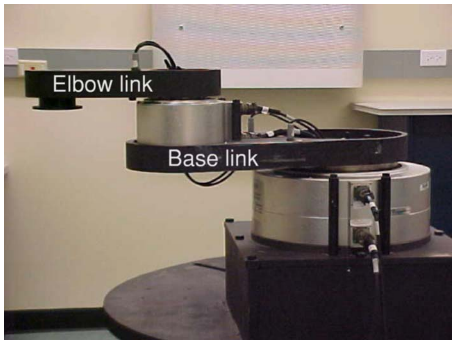

Fig. 1 Picture of the experimental platform

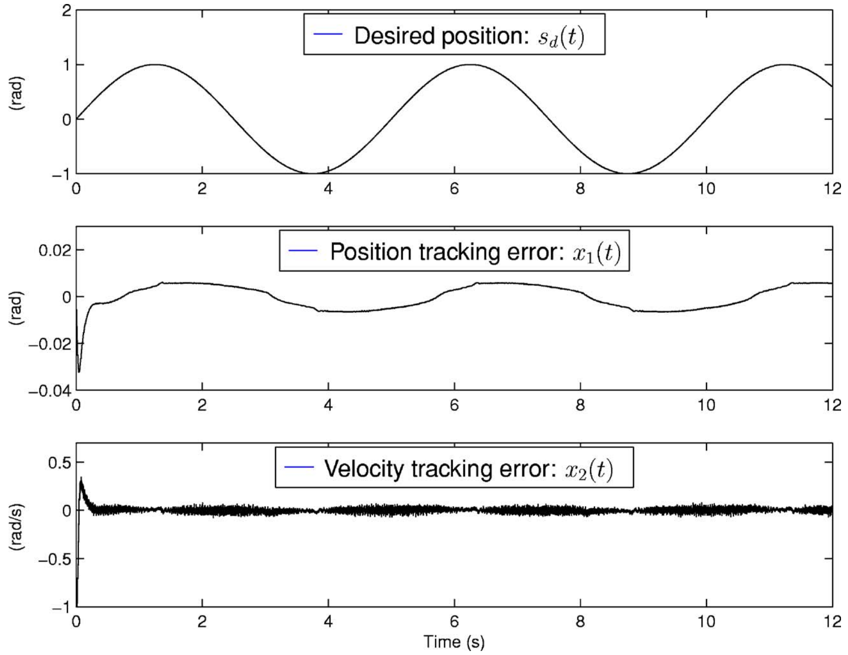

Fig. 2 Desired position and tracking errors

In the proposed design, if none of the elements of the vector $f_z(x)$ and the matrix $G_z(x)$ is zero, we require $m(p+1)$ filters for the estimation of the unmeasurable states and mp filters for the estimation of the unknown parameters.

Remark 3. The estimated parameters are not guaranteed to converge to their true values. Since the nonlinear system studied in this paper cannot be expressed in the form of a standard parametric model (see [16]), it is difficult to obtain the persistency of excitation conditions under which the estimated parameter $\hat{\Theta}$ converges to its true value Θ. To enhance the robustness of the closed-loop system due to parameter drift, we can use the σ-modification procedure given in [16]. In such a case, the parameter estimation algorithm (6) can be changed to

$$\dot{\hat{\Theta}} = -\sigma\Gamma\hat{\Theta} + 2\Gamma[G(x)\hat{Z} + f(x)]h^{\mathsf{T}}Px. \tag{15}$$

Choosing the same Lyapunov function candidate V as in (11), the time derivative of V becomes

$$\dot{V} \leqslant -x^{\mathsf{T}}Qx - \lambda_{\min}(\Lambda_{|\Theta|}Q_Z)\widetilde{Z}^{\mathsf{T}}\widetilde{Z} - 2\sigma\widetilde{\Theta}^{\mathsf{T}}\hat{\Theta}. \tag{16}$$

Since $-2\sigma\widetilde{\Theta}^{\mathsf{T}}\hat{\Theta} \leqslant -\sigma\widetilde{\Theta}^{\mathsf{T}}\widetilde{\Theta} + \sigma\|\Theta\|^2$, we have

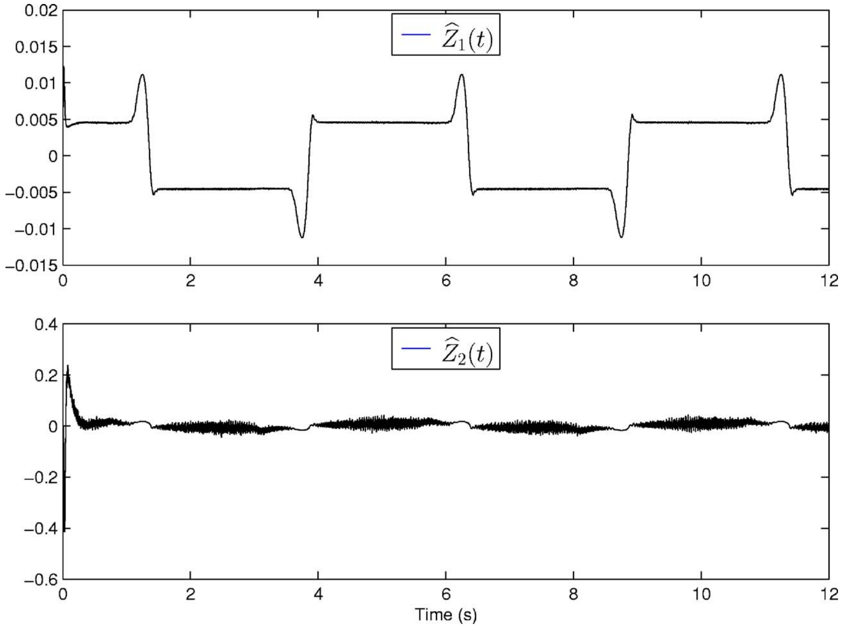

Fig. 3 Estimate of $Z(t)$: $\hat{Z}_1(t)$ and $\hat{Z}_2(t)$

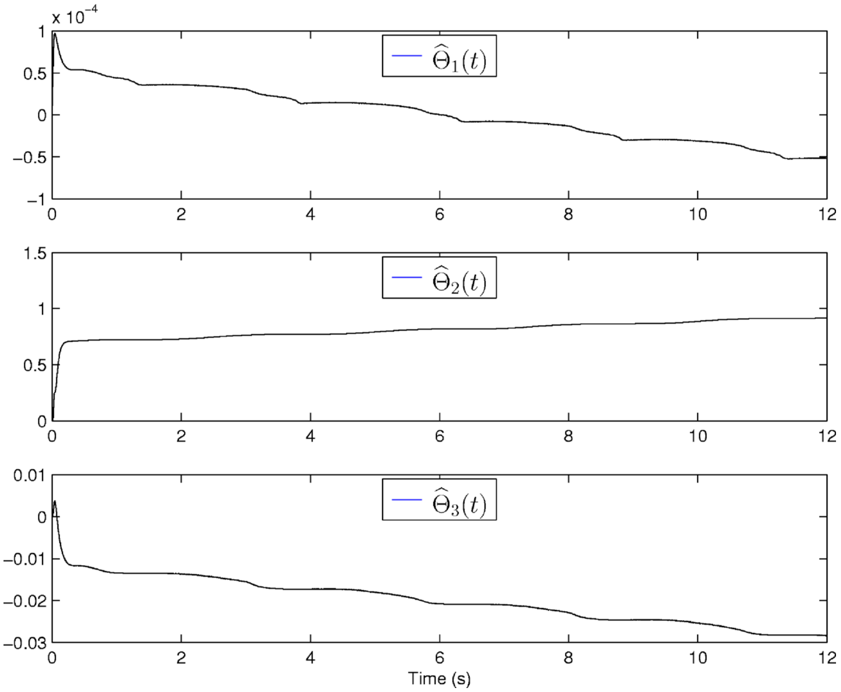

Fig. 4 Estimated parameters: $\hat{\Theta}_1(t)$, $\hat{\Theta}_2(t)$, and $\hat{\Theta}_3(t)$

$$\dot{V} \leqslant -\lambda_{\min}(Q)x^{\mathsf{T}}x - \lambda_{\min}(\Lambda_{|\Theta|}Q_Z)\widetilde{Z}^{\mathsf{T}}\widetilde{Z} - \sigma\widetilde{\Theta}^{\mathsf{T}}\widetilde{\Theta} + \sigma\|\Theta\|^2. \tag{17}$$

Since the parameter Θ is bounded, it follows that x, $\hat{\Theta}$, and Z converge to a residual set whose radius is proportional to the square root of the upper bound of $\sigma\|\Theta\|^2$.

5 Experimental Results

Consider a single-link mechanical system described by

$$J\ddot{s} = u - f_f \tag{18}$$

where J is the inertia of the link, s is the angular position of the link, $\dot{s}$ is the angular velocity of the link, u is the control input, and f_f is the friction torque described by the following LuGre dynamic friction model [17]:

$$\dot{z} = \dot{s} - \frac{\sigma|\dot{s}|}{g(\dot{s})}z, \tag{19}$$

$$f_f = \theta_1 z + \theta_2\dot{z} + \theta_3\dot{s}, \quad g(\dot{s}) = F_c + (F_s - F_c)e^{-(\dot{s}/\omega_s)^2} \tag{20}$$

where σ, θ_1, θ_2, θ_3, F_s, F_c, and ω_s are positive friction coefficients; σ, F_s, F_c, and ω_s are generally identified by experiments off-line and are assumed to be known for this simulation. J is known. The objective is to control the link such that the position and velocity of the link track a predefined trajectory s_d and $\dot{s}_d$, respectively. It is assumed that s_d and $\dot{s}_d$ are bounded, and the angular position and the angular velocity are measured.

Combining (18)–(20) and representing in matrix form yields

$$\dot{\zeta} = M\zeta + hu + h[f_\theta^{\mathsf{T}}\theta + \theta^{\mathsf{T}}G_z(\zeta)z], \tag{21a}$$

$$\dot{z} = a_z(\zeta) + B_z(\zeta)z \tag{21b}$$

where

$$\zeta = \begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix} = \begin{bmatrix} s\\ \dot{s}\end{bmatrix}, \quad M = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \quad f_\theta = \begin{bmatrix}0\\ -\zeta_2\\ -\zeta_2\end{bmatrix}, \quad h = \begin{bmatrix}0\\ \dfrac{1}{J}\end{bmatrix}$$

$$G_z(\zeta) = \begin{bmatrix}-1\\ \dfrac{\sigma|\zeta_2|}{g(\zeta_2)}\\ 0\end{bmatrix}, \quad \theta = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix}, \quad a_z(\zeta) = \zeta_2, \quad B_z(\zeta) = -\frac{\sigma|\zeta_2|}{g(\zeta_2)}.$$

Defining the trajectory vector $x_{\mathrm{d}}^{\mathsf{T}} = [s_d, \dot{s}_d]$ and representing (21*a*) and (21*b*) in terms of the tracking error $x := [x_1, x_2]^{\mathsf{T}} = \zeta - x_d$ results in the following error dynamics and z dynamics:

$$\dot{x} = Mx + hu + h[f_\theta^{\mathsf{T}}(x + x_d)\theta + \theta^{\mathsf{T}}G_z(x + x_d)z], \tag{22a}$$

$$\dot{z} = a_z(x + x_d) + B_z(x + x_d)z. \tag{22b}$$

The previous two equations can be rewritten in the following form suitable for the adaptive controller and observer design:

$$\dot{x} = Mx + hu + h[f^{\mathsf{T}}(x)\Theta + Z^{\mathsf{T}}G(x)\Theta], \tag{23a}$$

$$\dot{Z} = a(x) + B(x)Z \tag{23b}$$

where

$$\Theta \triangleq \begin{bmatrix}\Theta_1\\ \Theta_2\\ \Theta_3\end{bmatrix} = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix}, \quad Z \triangleq \begin{bmatrix}Z_1\\ Z_2\\ Z_3\end{bmatrix} = \begin{bmatrix}z\\ z\\ z\end{bmatrix}, \quad f(x) = \begin{bmatrix}0\\ -x_2 - \dot{s}_d\\ -x_2 - \dot{s}_d\end{bmatrix},$$

$$G(x) = \begin{bmatrix}-1 & 0 & 0\\ 0 & \dfrac{\sigma|x_2 + \dot{s}_d|}{g(x_2 + \dot{s}_d)} & 0\\ 0 & 0 & 0\end{bmatrix}, \quad a(x) = \begin{bmatrix}x_2 + \dot{s}_d\\ x_2 + \dot{s}_d\\ x_2 + \dot{s}_d\end{bmatrix},$$

$$B(x)=\begin{bmatrix} -\dfrac{\sigma|x_2+\dot{s}_d|}{g(x_2+\dot{s}_d)} & 0 & 0 \\ 0 & -\dfrac{\sigma|x_2+\dot{s}_d|}{g(x_2+\dot{s}_d)} & 0 \\ 0 & 0 & -\dfrac{\sigma|x_2+\dot{s}_d|}{g(x_2+\dot{s}_d)} \end{bmatrix}.$$

Notice that $x(=\zeta - x_d)$ is available because ζ is measurable and x_d is known; and Θ and Z are estimated; Z_3 need not be estimated because $g_{33}(x)=0$.

Experiments were conducted on the base link of a two-link NSK manipulator shown in Fig. 1. The base link is controlled to follow a sinusoidal trajectory, $s_d(t)=\sin(0.4\pi t)$. The inertia of the base link is $J=3.4$. The following values are chosen: $c^{\mathsf{T}}=[2500,100]*3.4$, $\Gamma=\mathrm{diag}(5,5,5)$. $P_z=0.1I$,

$$P=\begin{bmatrix} 626.25 & 0.01 \\ 0.01 & 0.2501 \end{bmatrix}.$$

The parameters in the simulation are: $\sigma=340$, $F_s=11$, $F_c=1.557$, $\omega_s=0.14$. The following initial values are chosen: $\hat{\Theta}^{\mathsf{T}}(0)=[0,0,0]$, and $\hat{Z}^{\mathsf{T}}(0)=[0,0,0]$. Experimental results are shown in Figs. 2–4. Figure 2 shows the desired position trajectory, position tracking error, and velocity tracking error from the top plot to the bottom plot, respectively. The estimates of the unmeasured state $Z(t)$, $\hat{Z}_1(t)$, and $\hat{Z}_2(t)$, are shown in Fig. 3. Parameter estimates are shown in Fig. 4.

6 Conclusions

A new adaptive controller and a nonlinear observer are designed for a class of nonlinear systems that contain the products of an unmeasured state and an unknown parameter. The proposed design is validated by experimental results on a single-link mechanical system with dynamic friction. Future work should focus on the inclusion of coupled terms of the unknown parameters and unmeasured states in the unmeasurable state dynamics. Future research should also focus on the investigation of the existence of parameter independent state diffeomorphisms that will transform a general nonlinear systems to the class of systems considered in this paper.

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